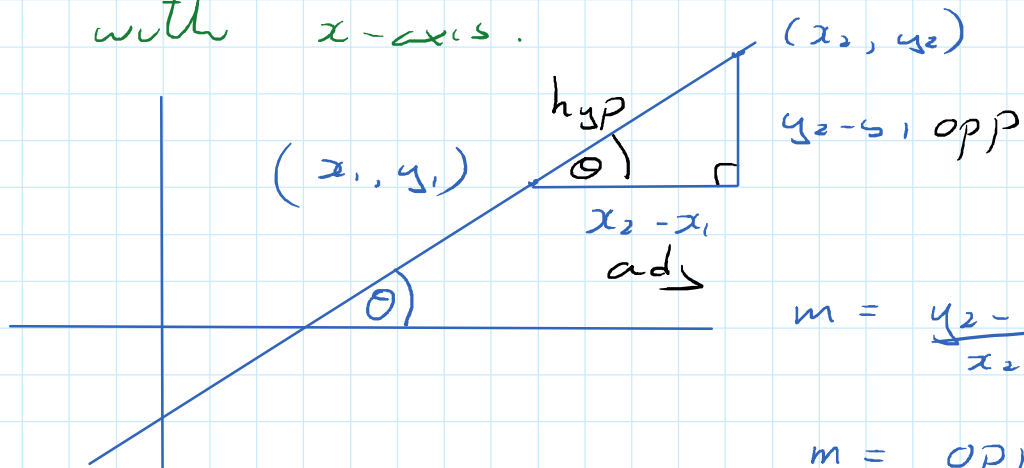


## Angle Between 2 Lines

Angles with x-axis.



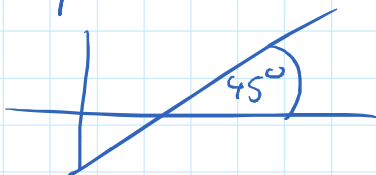
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\text{opp}}{\text{adj}} = \tan \theta$$

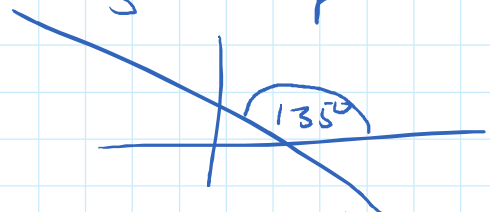
$$m = \tan \theta$$

$\theta$  is the angle formed with positive sense of x-axis.

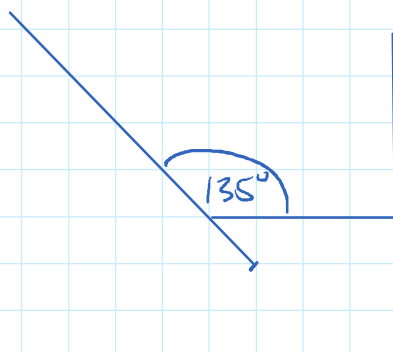
Angle of  $45^\circ$



Angle of  $135^\circ$



Find equation of line through  $(-3, 1)$  which cuts positive sense of x-axis at  $135^\circ$ .



$$m = \tan 135^\circ = -1$$

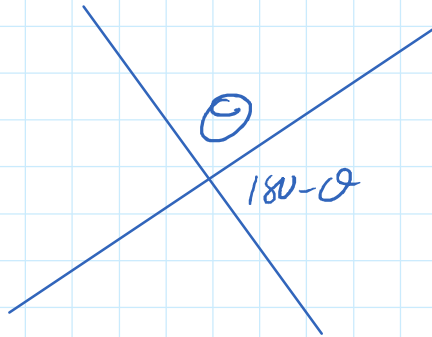
$$y - y_1 = m(x - x_1)$$

$$y - 1 = -1(x + 3)$$

$$y - 1 = -x - 3$$

$$x + y = -2$$

Angle between 2 lines.



$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

$m_1$  = slope of first line

$m_2$  = slope of second line

$$m_1 = m_2 \Rightarrow \theta = 0 \Rightarrow \text{parallel}$$

$$m_1 m_2 = -1 \Rightarrow 1 + m_1 m_2 = 0 \Rightarrow \tan \theta = \text{undefined}$$
$$\theta = 90^\circ$$

Find the acute angle between  
 $2x + y = 5$  and  $3x - y = 7$ .

$$y = -2x + 5$$

$$m_1 = -2$$

$$-y = -3x + 7$$

$$y = 3x - 7$$

$$m_2 = 3$$

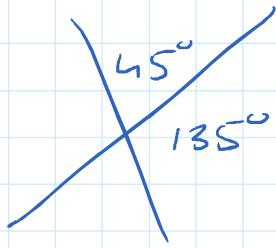
$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan \theta = \pm \frac{-2 - 3}{1 - 2(3)}$$

$$\tan \theta = \pm (1)$$

$$\tan \theta = 1$$

$$\theta = 45^\circ$$



$$\tan(\text{acute}) > 0$$

$$\tan(\text{obtuse}) < 0$$

Find obtuse angle between  
 $2x - 3y = 5$  and  $3x + 5y = 7$ .

$$2x - 3y = 5$$

$$-3y = -2x + 5$$

$$3y = 2x - 5$$

$$y = \frac{2}{3}x - \frac{5}{3}$$

$$m_1 = \frac{2}{3}$$

$$m_2 = -\frac{3}{5}$$

$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

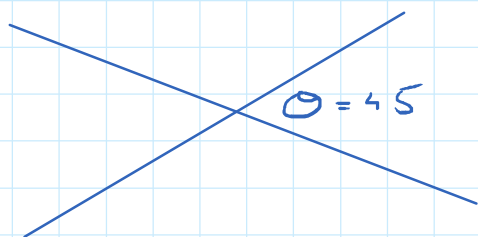
$$\tan \theta = \pm \frac{\frac{2}{3} + \frac{3}{5}}{1 - \frac{2}{3} \left( \frac{3}{5} \right)}$$

$$\tan \theta = \frac{19}{9}$$

$$\theta = 64.65$$

Answer  $180 - 64.65 = 115.35^\circ$

Find 2 lines through  $(-1, 3)$  which cut  $2x + y = 7$  at  $45^\circ$ .



$$2x + y = 7$$

$$y = -2x + 7$$

$$m_1 = -2.$$

$$m_2 = ?$$

$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan 45 = \pm \left( \frac{-2 - m}{1 - 2m} \right)$$

$$1 = \pm \left( \frac{-2 - m}{1 - 2m} \right)$$

$$1 - 2m = \pm (-2 - m)$$

$$1 - 2m = -2 - m \quad \text{or} \quad 1 - 2m = -(-2 - m)$$

$$-m = -3$$

$$m = 3$$

$$1 - 2m = 2 + m$$

$$-3m = 1$$

$$m = -\frac{1}{3}$$

$$y - 3 = 3(x - 1)$$

$$y - 3 = -\frac{1}{3}(x - 1)$$