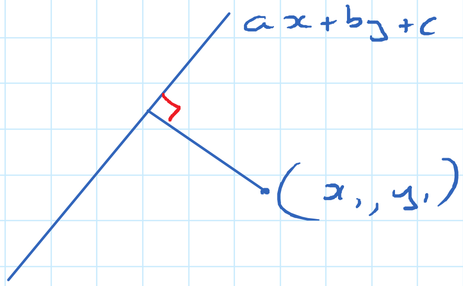
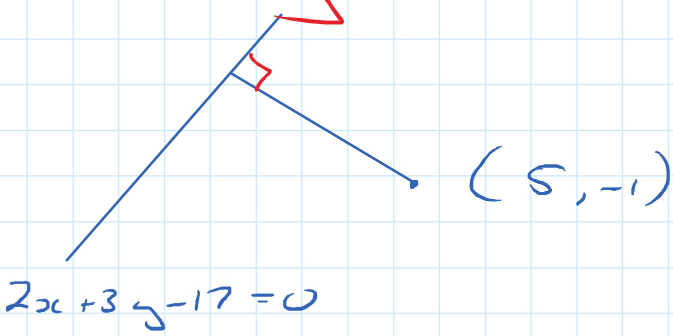
This is the distance from a point (x_1, y_1) to a line $ax + by + c = 0$



$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Perpendicular is the shortest distance.

Find the distance from $(5, -1)$ to $2x + 3y = 17$



$a = 2 \quad b = 3 \quad c = -17$

$x_1 = 5 \quad y_1 = -1$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\frac{|2(5) + 3(-1) - 17|}{\sqrt{2^2 + 3^2}} = \frac{|-10|}{\sqrt{13}}$$

$\frac{10}{\sqrt{13}}$ units.

Note The answer of $\frac{|-10|}{\sqrt{13}}$ means we are on negative side of line.

Find distance from $(-5,-1)$ to $3x-2y-1=0$

$a=3 \quad b=-2 \quad c=-1 \quad x_1=-5 \quad y_1=-1$

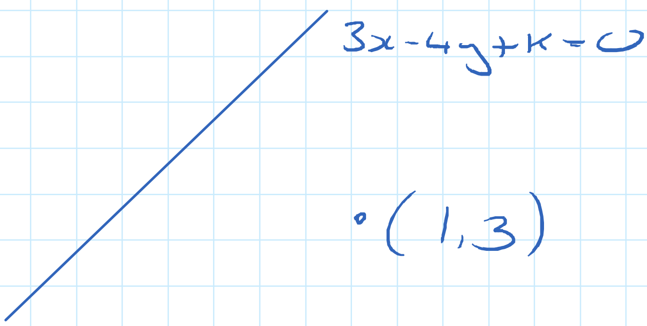
$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$\frac{|3(-5)-2(-1)-1|}{\sqrt{3^2+(-2)^2}} = \frac{|-14|}{\sqrt{13}}$$

$$= \frac{14}{\sqrt{13}} \text{ units}$$

$$= \frac{14}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{14\sqrt{13}}{13}$$

The distance from $3x-4y+k=0$ to $(1,3)$ is 6 units find 2 values of k?



$a=3 \quad b=-4 \quad c=k$

$x_1=1 \quad y_1=3$

Distance $=6$

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$\frac{|3(1)-4(3)+k|}{\sqrt{3^2+(-4)^2}} = 6$$

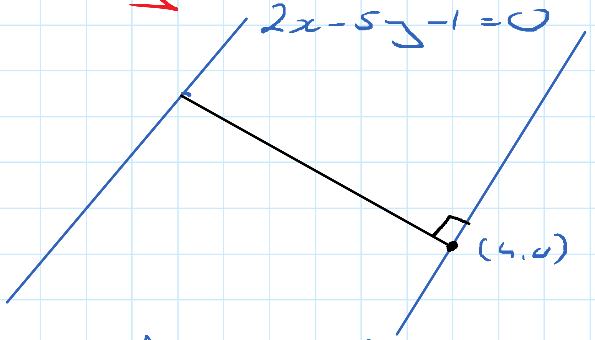
$$\frac{|3-12+k|}{\sqrt{9+16}} = 6$$

$$|k-9| = 5(6)$$

$k - 9 = 30$ $\qquad$ $k - 9 = -30$

$k = 39$ $\qquad$ $k = -21$

Find the distance from $2x - 5y - 1 = 0$ to $2x - 5y - 8 = 0$



Parallel Lines.

$y = 0 \quad x = 4$

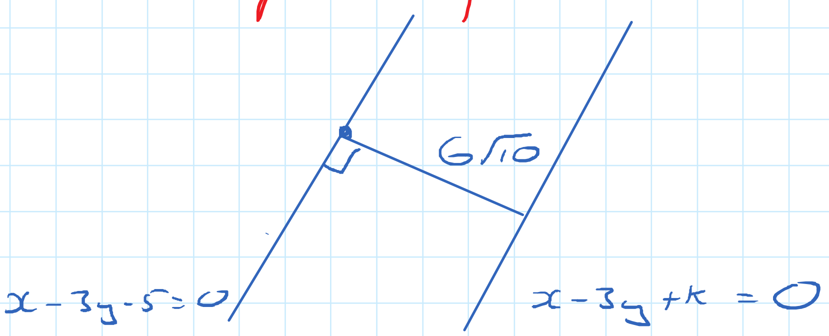
Distance from $(4, 0)$ to $2x - 5y - 1 = 0$

$a = 2 \quad b = -5 \quad c = -1 \quad x_1 = 4 \quad y_1 = 0$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\frac{|2(4) - 5(0) - 1|}{\sqrt{2^2 + (-5)^2}} = \frac{7}{\sqrt{29}} \text{ units}$$

The line p is $x - 3y - 5 = 0$. Find 2 lines parallel to p which are $6\sqrt{10}$ units from p.



$y = 0 \quad x = 5$

$(5, 0)$ to $x - 3y + k = 0$ Distance

is $6\sqrt{10}$

$a = 1 \quad b = -3 \quad c = k \quad x_1 = 5 \quad y_1 = 0$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

$$\frac{|1(5) - 3(0) + k|}{\sqrt{1^2+(-3)^2}} = 6\sqrt{10}$$

$$\frac{|k+5|}{\sqrt{10}} = 6\sqrt{10}$$

$$|k+5| = 60$$

$k + 5 = 60 \qquad k + 5 = -60$

$k = 55 \qquad k = -65$

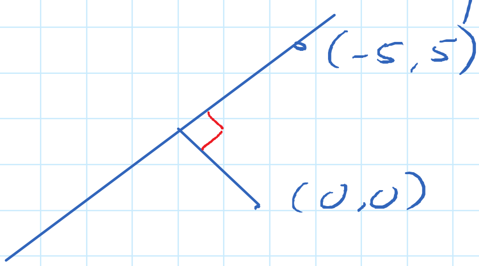
$x - 3y + 55 = 0$

$x - 3y - 65 = 0$

Find 2 lines through $(-5,5)$ which are a distance of $\sqrt{5}$ from origin.

Need equation of a line

(Never look for 2 find 1 and second will appear).



$y - y_1 = m(x - x_1)$

$y - 5 = m(x + 5)$

$y - 5 = mx + 5m$

$mx - y + 5 + 5m = 0$

Distance from $mx - y + 5 + 5m = 0$ to $(0,0)$ is $\sqrt{5}$.

$a = m \quad b = -1 \quad c = 5 + 5m \quad x_1 = 0 \quad y_1 = 0$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\frac{|5 + 5m|}{\sqrt{m^2+1}} = \sqrt{5}$$

$$|5 + 5m| = \sqrt{5}\sqrt{m^2+1}$$

$$25 + 50m + 25m^2 = 5(m^2+1)$$

$$25m^2 + 50m + 25 = 5m^2 + 5$$

$$20m^2 + 50m + 20 = 0$$

$$2m^2 + 5m + 2 = 0$$

$$(2m+1)(m+2) = 0$$

$$m = -\frac{1}{2} \qquad m = -2$$

$$y - 5 = -\frac{1}{2}(x+5) \qquad y - 5 = -2(x+5)$$