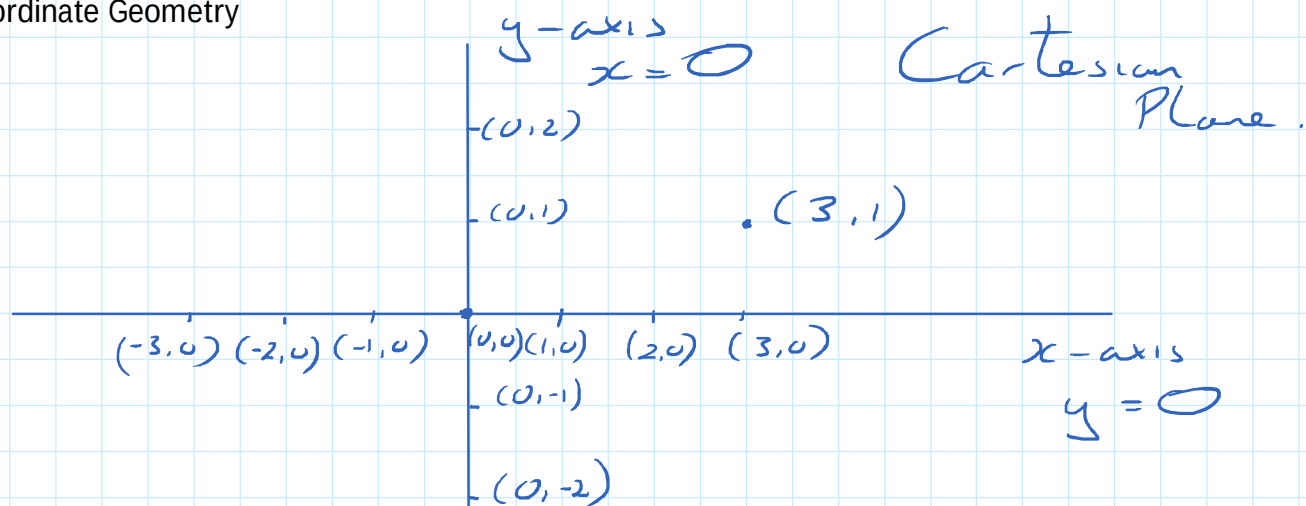
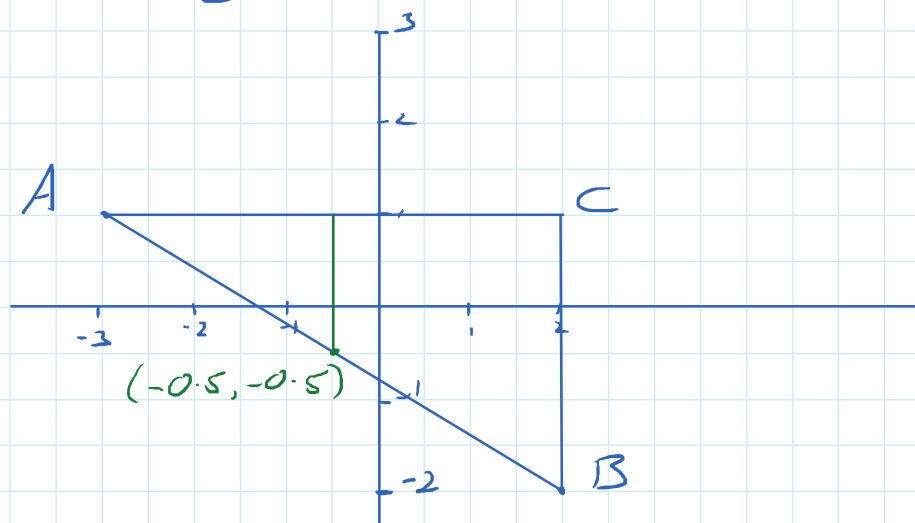




$(0,0)$ is origin. Start at origin
go x first (left or right) then
 y (up or down)
Every point is (x,y)

On a Cartesian Plane plot
 $A(-3,1)$ and $B(2,-2)$.

Note. Points are denoted by Capital letters.
Lines by lower case letters.



Use the graph to find

(i) Midpoint of AB

(ii) Slope of AB

(iii) $|AB|$.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

Slope = angle = gradient
= incline.
= how steep.

$$= \frac{\text{change in } y}{\text{change in } x} = \text{rate of change.}$$

Rate of change

Height $\rightarrow$ Time

Age $\rightarrow$ Time = constant

Weight $\rightarrow$ Food

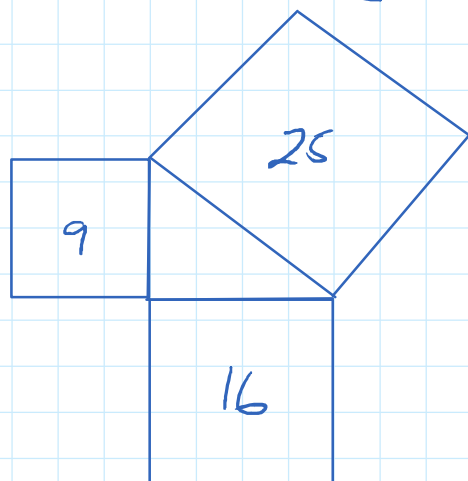
Knowledge $\rightarrow$ Time.

$$m = -\frac{3}{5} \quad \text{go down 3 right 5.}$$

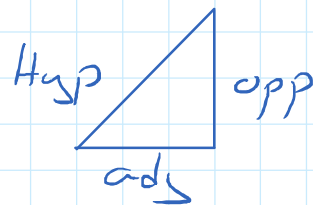
Down $\Rightarrow$ negative slope

Up $\Rightarrow$ positive slope.

Distance = Pythagoras' Theorem.

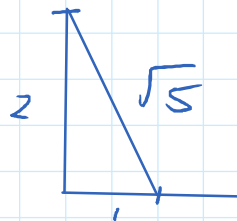
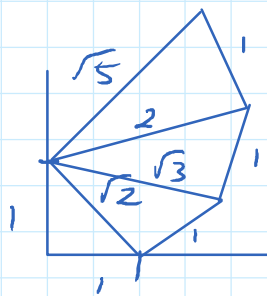


$$3^2 + 4^2 = 5^2$$



$$H^2 = O^2 + A^2$$

Draw $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$



$$|AB|^2 = |AC|^2 + |CB|^2 \quad \text{Question}$$

$$|AB|^2 = 5^2 + 3^2$$

$$|AB| = \sqrt{34}$$

Note: $|AB|$ = distance from A to B.

Midpoint = half way = mean

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

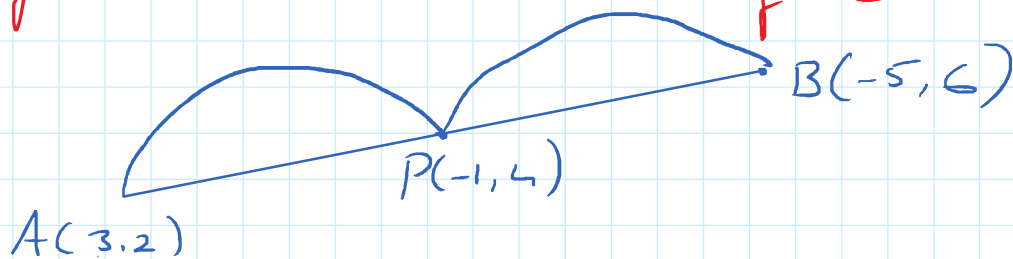
Find midpoint of $A(-3, 5)$ and $B(1, -10)$

$$\begin{matrix} (-3, 5) & (1, -10) \\ x_1, y_1 & x_2, y_2 \end{matrix}$$

$$\text{Midpoint} \left(\frac{-3+1}{2}, \frac{5-10}{2} \right)$$

$$\left(-1, -\frac{5}{2} \right)$$

$P(-1, 4)$ is midpoint of $A(3, 2)$ and B
find co-ordinates of B .



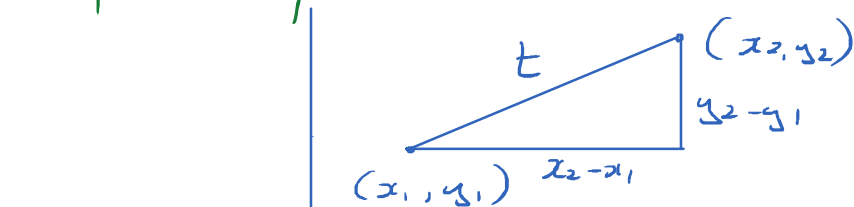
Central symmetry

$$A(3, 2)$$

$$P \begin{matrix} -4+2 \\ (-1, 4) \\ -4+2 \end{matrix}$$

$$B(-5, 6)$$

Distance / Slope



$$t^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$t = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$A(-3, 1)$ and $B(2, -5)$ are 2 points
find $|AB|$

$$\begin{array}{cc} (-3, 1) & (2, -5) \\ x_1, y_1 & x_2, y_2 \end{array}$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(2 + 3)^2 + (-5 - 1)^2}$$

$$\sqrt{5^2 + (-6)^2} = \sqrt{61} \text{ units.}$$

$A(3, -1)$ and $B(k, 3)$ are 2 points such that $|AB| = 5$ units find 2 values for k

$$\begin{array}{cc} (3, -1) & (k, 3) \\ x_1, y_1 & x_2, y_2 \end{array}$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(k - 3)^2 + (3 + 1)^2} = 5$$

$$(k - 3)^2 + 4^2 = 5^2$$

$$(k - 3)^2 + 16 = 25$$

$$(k - 3)^2 = 9$$

$$k - 3 = 3 \quad \text{or} \quad k - 3 = -3$$

$$k = 6$$

$$k = 0$$

Find slope of $(3, -1)$ to $(-2, 4)$

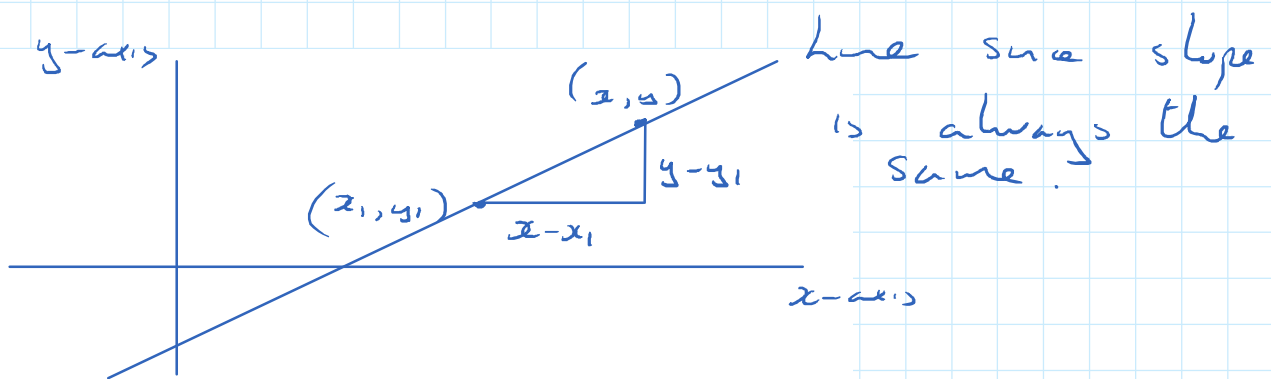
$$\begin{matrix} (3, -1) & (-2, 4) \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{-2 - 3} = -1$$

Equation of a line

Why is a line a line? (not a curve)

It is straight.



$$\frac{y - y_1}{x - x_1} = m$$

$$y - y_1 = m(x - x_1)$$

Equation of a line

Find equation of line through $(-1, 5)$ and $(2, 3)$

$$\begin{matrix} (-1, 5) & (2, 3) \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

$$m = \frac{3 - 5}{2 - (-1)} = -\frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{2}{3}(x + 1)$$

$$3y - 15 = -2x - 2$$

$$2x + 3y = 13$$

An plane, on way to Russia follows a flight path. This is a linear path from 2 to 15 minutes. After 2 minutes it is 2000 ft and 10 minutes 30,000 ft. Find the path of the flight?

(x, y) x = independent event

y = dependent event.

y depends on x .

$$\begin{pmatrix} 2, & 2000 \\ x_1 & y_1 \end{pmatrix}$$

$$\begin{pmatrix} 10, & 30000 \\ x_2 & y_2 \end{pmatrix}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30000 - 2000}{10 - 2}$$

$$m = \frac{28000}{8} = 3500$$

Plane goes up 3,500ft every minute.

$$(2, 2000)$$

$$m = 3500$$

$$y - 2000 = 3500(x - 2)$$

$$y - 2000 = 3500x - 7000$$

$$3500x - y = 5000$$

To find slope of a line

Find slope of $2x + 3y = 6$

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = mx - mx_1$$

$$y = mx - mx_1 + y_1$$

$$y = mx + c \quad m = \text{before the } x$$

$$2x + 3y = 6$$

$$3y = -2x + 6$$

$$y = \frac{-2x + 6}{3}$$

$$y = \frac{-2x}{3} + \frac{6}{3}$$

$$m = -\frac{2}{3}$$

Find slope of $5x - 7y = 11$

$$-7y = -5x + 11$$

$$7y = 5x - 11$$

$$y = \frac{5}{7}x - \frac{11}{7}$$

$$m = \frac{5}{7}$$

$$y\text{-axis } x=0$$

$$\left(0, -\frac{11}{7}\right)$$

To Draw a Line

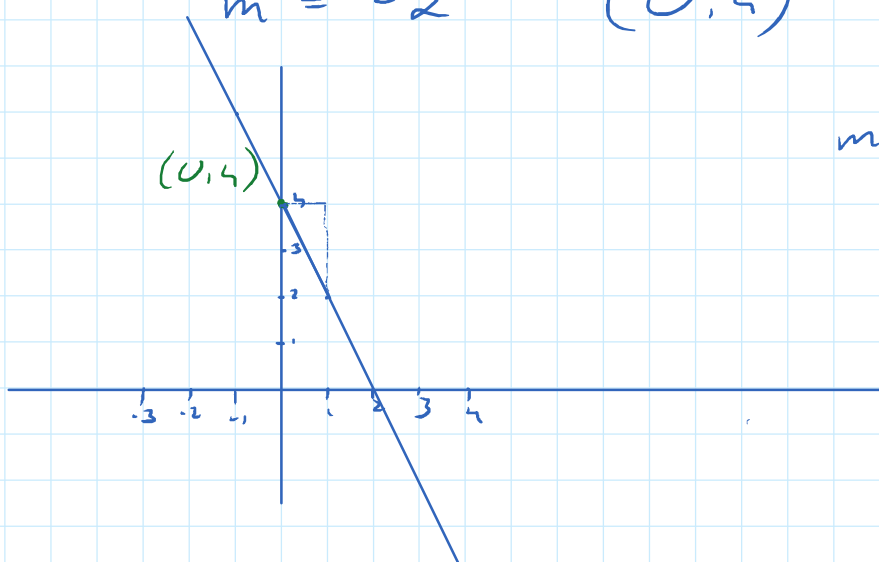
Draw

$$2x + y = 4$$

$$2x + y = 4$$

$$y = -2x + 4$$

$$m = -2 \quad (0, 4)$$



$$m = -\frac{2}{1}$$

Draw

$$2x - 3y = 9$$

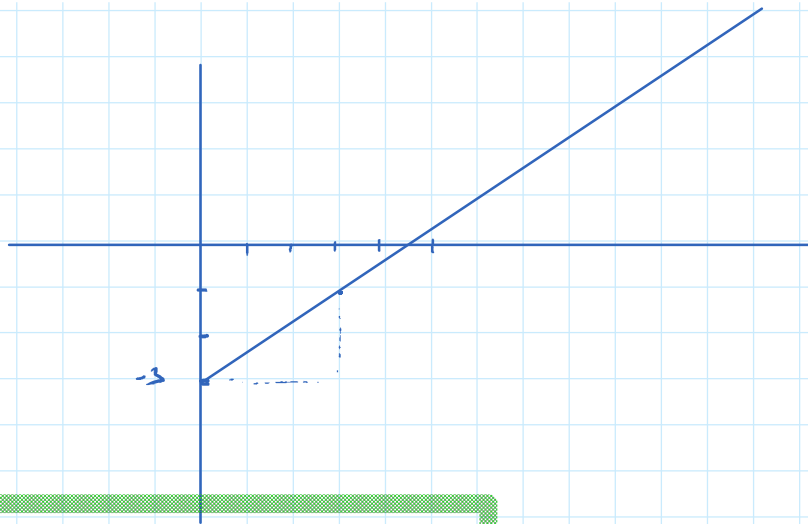
$$2x - 3y = 9$$

$$-3y = -2x + 9$$

$$3y = 2x - 9$$

$$y = \frac{2x - 9}{3}$$

$$(0, -3) \quad m = \frac{2}{3}$$



x-axis	$y = 0$
y-axis	$x = 0$

Parallel and Perpendicular Lines

Parallel $\parallel$ = same slope $\Rightarrow m_1 = m_2$

Perpendicular $\perp$ = invert and change the sign of slopes $\Rightarrow m_1 m_2 = -1$

Slope	Parallel	Perpendicular
$-\frac{3}{7}$	$-\frac{3}{7}$	$\frac{7}{3}$
$-\frac{1}{9}$	$-\frac{1}{9}$	9
10	10	$-\frac{1}{10}$

Find equation of line through origin parallel to $5x + 3y = 6$

$$5x + 3y = 6$$

$$3y = -5x + 6$$

$$y = -\frac{5}{3}x + 2$$

$$m = -\frac{5}{3}$$

Required $m = -\frac{5}{3}$ Point $(0, 0)$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{5}{3}(x - 0)$$

$$3y = -5x$$

$$5x + 3y = 0$$

Parallel Lines

$$ax + by = c$$

$$ax + by = d$$

Alternative method

Answer

$$5x + 3y = K \quad (0, 0)$$

$$x = 0 \quad y = 0$$

$$5(0) + 3(0) = K$$

$$K = 0$$

Answer

$$5x + 3y = 0$$

If a point is on a line substitution.

Perpendicular Line

$$ax + by = c$$

$$m = -\frac{a}{b}$$

$$bx - ay = d$$

$$m = \frac{b}{a}$$

Find line through $(1, -5)$ which is perpendicular to $3x + 7y = 11$.

Answer of form

$$7x - 3y = k$$

$$x = 1 \quad y = -5$$

$$7 + 15 = k$$

$$7x - 3y = 22$$

Find line through $(-1, -6)$ perpendicular to $x - 5y = 10$

Answer

$$5x + y = k$$

$$5(-1) - 6 = k$$

$$5x + y = -11$$

To find where 2 lines meet.

p is the line $2x + 3y = 8$ and q is $x + 2y = 5$. Find A if $A \in (p \cap q)$

$p \cap q = p$ intersection q .

= meet = cut = cross

Use simultaneous Equations

$$x=1 \quad x=2.$$

Transformation.

Translation = movement in a certain direction for a certain distance.

Find image of $(3,1)$ under translation $(-1,5)$ to $(4,7)$

$$(-1,5)$$

$$+5+2$$

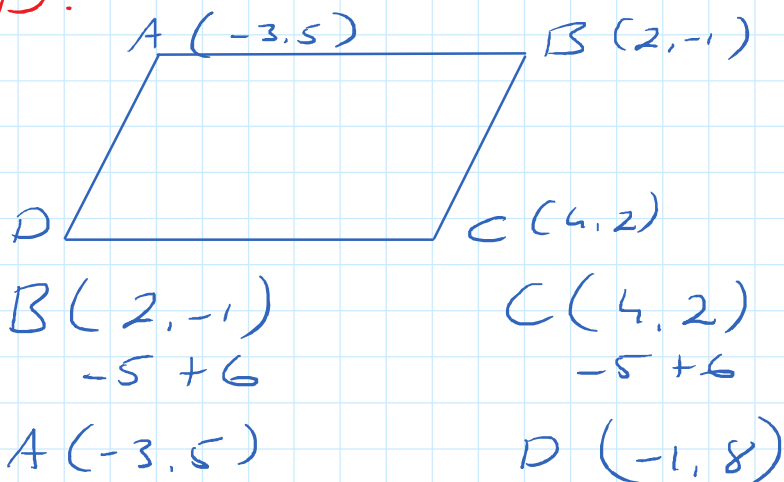
$$(4,7)$$

$$(3,1)$$

$$+5+2$$

$$(8,3)$$

$A(-3,5)$, $B(2,-1)$, $C(4,2)$ are 3 points in parallelogram ABCD.
Find D.



$$B(2,-1)$$

$$-5+6$$

$$A(-3,5)$$

$$C(4,2)$$

$$-5+6$$

$$D(-1,8)$$

Central symmetry
Move through a point and go same

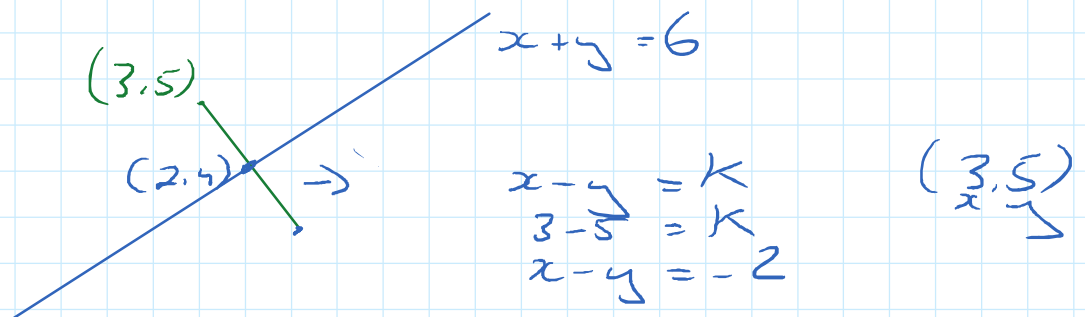
again the other side.

Find image of $(2, 1)$ under central symmetry in $(-1, 3)$.

$$\begin{array}{r} (2, 1) \\ -3 \quad +2 \\ \hline (-1, 3) \\ -3 \quad +2 \\ \hline (-4, 5) \end{array}$$

Axial symmetry - go through a line at right angles and go same again the other side.

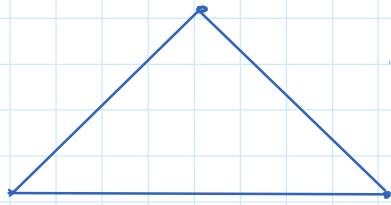
Find axial symmetry of $(3, 5)$ under $x + y = 6$



$$\begin{array}{r} x + y = 6 \\ x - y = -2 \\ \hline 2x = 4 \\ x = 2 \\ y = 4 \end{array}$$

$$\begin{array}{r} (3, 5) \\ -1 \quad -1 \\ \hline (2, 4) \\ -1 \quad -1 \\ \hline (1, 3) \end{array}$$

Area of a Triangle



Vertices = corner points

Vertex = point.

$$A = \frac{1}{2} |x_1 y_2 - x_2 y_1| \text{ one point must be } (0,0)$$

Find area of $(3, -1)$, $(-6, 4)$ and $(0, 0)$

$$\begin{array}{ccc} (3, -1) & (2, 5) & (-6, 4) \\ -3 & +1 & -3 & +1 \\ (0, 0) & (-1, 6) & (-9, 5) \\ & x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\begin{aligned} A &= \frac{1}{2} |x_1 y_2 - x_2 y_1| \\ &= \frac{1}{2} |-1(5) + 9(6)| \\ &= \frac{1}{2} |49| = \frac{49}{2} \text{ sq units.} \end{aligned}$$

$A(5, -2)$, $B(-1, 6)$ and $C(3, k)$ form a triangle of area 12 sq units. Find k .

$$\begin{array}{ccc} (5, -2) & (-1, 6) & (3, k) \\ -5 & +2 & -5 & +2 \\ (0, 0) & (-6, 8) & (-2, k+2) \\ & x_1 & y_1 & x_2 & y_2 \end{array}$$
$$A = \frac{1}{2} |x_1 y_2 - x_2 y_1|$$

$$= \frac{1}{2} | -6(k+2) + 2(8) | = 12$$

$$\frac{1}{2} | -6k - 12 + 16 | = 12$$

$$\frac{1}{2} | -6k + 4 | = 12$$

$$| -3k + 2 | = 12$$

$$-3k + 2 = 12$$

$$-3k = 10$$

$$k = -\frac{10}{3}$$

$$-3k + 2 = -12$$

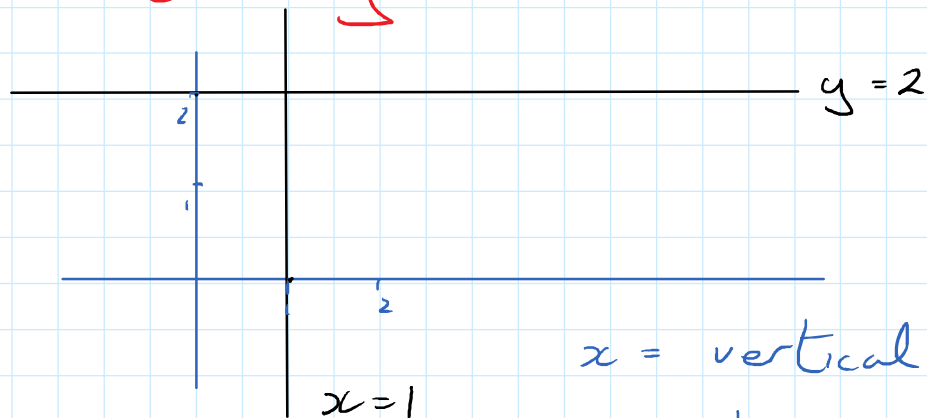
$$-3k = -14$$

$$k = \frac{14}{3}$$

Vertical and Horizontal Lines.

On a Cartesian Plane plot

$x=1$ and $y=2$.



x = vertical

y = horizontal.

Note: Collinear $\Rightarrow$ 3 or more points on same line.

Slope AB = slope BC