Series.

Prove $\sum_{n=1}^{n} n = \frac{n}{2}(n+1)$

$1 + 2 + 3 + \cdots + n = \frac{n}{2}(n+1)$

LHS = series RHS = algebra.

Prove for $n = 1$

LHS we have 1 term. RHS sub $n=1$

$1 = \frac{1}{2}(1+1)$

$1 = 1$ true

Assume for $n = k$

LHS we have k terms. RHS sub in $n = k$

$\boxed{1 + 2 + \cdots + k} = \boxed{\frac{k}{2}(k+1)}$

Prove $n = k+1$

LHS we have $k+1$ terms (extra term). RHS sub in

$\boxed{\underbrace{1 + 2 + \cdots + k}_{\text{Assumption}}} + k+1 = \frac{k+1}{2}(k+1+1)$

replace with $\frac{k}{2}(k+1)$

$\boxed{\frac{k}{2}(k+1)} + \frac{k+1}{1}$ Algebra now on LHS

$\frac{k(k+1) + 2(k+1)}{2}$

$\frac{k+1}{2}(k+2) = \frac{k+1}{2}(k+2)$

EKKER

$$\sum_{n=1}^{n}(2n-1) = n^2$$

$$1 + 3 + 5 + \cdots 2n-1 = n^2$$

Prove $n=1$

$$1 = 1^2 \quad \text{true}$$

Assume $n=k$

$$\boxed{1+3+5+\cdots+2k-1} = k^2$$

Prove $n=k+1$

$$\boxed{1+3+5+\cdots+2k-1} + 2(k+1)-1 = (k+1)^2$$

$$k^2 + 2k+2-1 = k^2+2k+1$$

$$k^2+2k+1 = k^2+2k+1$$

$$\sum_{n=1}^{n} n^2 = \frac{n}{6}(n+1)(2n+1), \quad n \in N.$$

$$\sum_{n=1}^{n} n^2 = 1^2+2^2+3^2+\cdots n^2 = \frac{n}{6}(n+1)(2n+1)$$

Prove $n=1$

$$1 = \frac{1}{6}(2)(3) \quad \text{True}$$

Assume $n=k$

$$\boxed{1^2+2^2+\cdots+k^2} = \frac{k}{6}(k+1)(2k+1)$$

Prove $n=k+1$

$$\boxed{1^2+2^2+\cdots+k^2} + (k+1)^2 = \frac{k+1}{6}(k+1+1)(2(k+1)+1)$$

$$\frac{k}{6}(k+1)(2k+1) + \left(\frac{k+1}{1}\right)^2 = \frac{k+1}{6}(k+2)(2k+3)$$

$$\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$\frac{k+1}{6}(2k^2+k+6k+6)$$

$$\frac{k+1}{6}(2k^2+7k+6)$$

$$\frac{k+1}{6}\,(k+2)\,(2k+3)$$

Conclusion $\Rightarrow$