

# Inequalities.

Prove  $n! > 2^n$ ,  $n \geq 4$   $n \in \mathbb{N}$ .

Prove  $n = 4$

$$4! > 2^4$$

$$24 > 16$$

True

Assume  $n = k$

$$k! > 2^k$$

Factorial

Prove  $n = k+1$   $(k+1)! > 2^{k+1}$

$$(k+1)k! > 2(2^k)$$

bigger one  
down  $\Rightarrow$

combinatorics

notes.

Multiplication both sides  
and assumption.

$$k+1 > 2 \quad k \geq 4$$

$k! > 2^k$  from assumption.

Conclusion - Since true for  $n = 4$  and  
 $n = k+1$  true for all  $n \geq 4$

Prove  $3^n > 2n+1$   $n > 1$ ,  $n \in \mathbb{N}$

Prove  $n = 2 \Rightarrow 3^2 > 2(2) + 1$

$$9 > 5$$

True

Assume  $n = k$

$$3^k > 2k + 1$$

Prove  $n = k+1$

$$3^{k+1} > 2(k+1) + 1$$

$$3 \cdot 3^k > 2k + 2 + 1$$

Need the

assumption

LHS has multiply, but right has adding  
Create addition on left.

$$(2+1)3^k > 2k+1+2$$

$$2(3^k) + 3^k > 2k+1+2$$

$$3^k > 2k+1 \quad \text{assumption}$$

Conclusion  $2(3^k) > 2$  since  $k > 1$   
 $\Rightarrow$  .