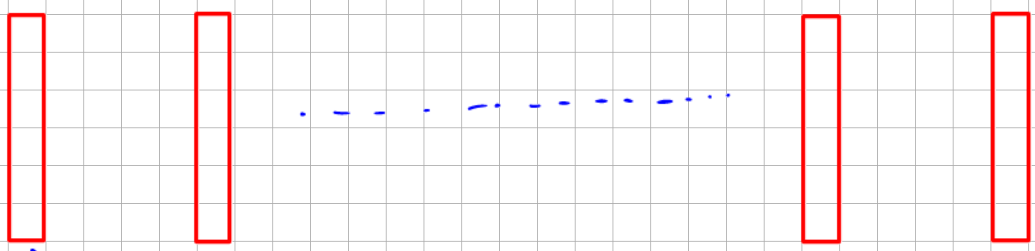
Proof by Induction.

Prove $6^n - 1$ is divisible by 5.



Test first one
Prove for $n=1$

Assume for $n=k$
any term

Prove for $n=k+1$
next one

Prove for $n = 1$

Assume for $n = k$

Prove for $n = k+1$

Conclusion = since true $n=1$ and $n=k+1$ then true for all n.

Write down conclusion no matter what.

Prove $6^n - 1$ is divisible by 5.

Prove $n=1$
$6-1 = 5 \quad \div 5 = 1$ true.

Assume for $n=k$
$\boxed{6^k - 1}$ is divisible by 5.

Prove for $n = k+1$

$6^{k+1} - 1$ $\qquad a^{p+q} = a^p . a^q$

$6(6^k) - 1$ $\qquad$ Hint $1+2=3$

$(5+1)\, 6^k - 1$ $\qquad 4+5=9$

$\qquad 6+7=13$

$\qquad 6 = 4+2$

$5(6^k)\boxed{+\,6^k - 1}$ $\qquad 6 = 5+1$

$5(6^k)$ true because of 5 $\qquad 6^k - 1$ true from assumption.

Conclusion : Since true for $n=1$ and $n=k+1$ then true for all n

Need to get back to the assumption.

Prove $8^n - 3^n$ is divisible by 5.

Prove $n = 1$

$8 - 3 = 5 \quad \div 5 = 1$ True

Assume $n = k$ $\quad 8^k - 3^k$ is divisible by 5

Prove $n = k+1$

$8^{k+1} - 3^{k+1}$

$8.8^k - 3.3^k$

$(5+3)\,8^k - 3.3^k$

$5(8^k) + 3(8^k - 3^k)$

True $5(8^k)$ is divisible 5

$3(8^k - 3^k)$ true from assumption.

Prove $3^{2n} - 1$ is divisible by 8.

Prove $n = 1$

$9 - 1 = 8 \div 8 = 1$ True

Assume $n = k$

$3^{2k} - 1$ is divisible by 8

Prove $n = k+1$

$3^{2k+2} - 1$ is divisible by 8

This is not important → $3^2.3^{2k} - 1$ ← did anything new appear. Take 9 infront and split it

$(8+1)\,3^{2k} - 1$

$8(3^{2k}) + 3^{2k} - 1$ ← back to assumption.

$8(3^{2k})$ true cause of 8

$3^{2k} - 1$ true assumption

Prove $9^n - 3^n$ is divisible by 6.

$n = 1$ $\quad 9 - 3 = 6$ true

$n = k$ $\quad 9^k - 3^k$

$n = k+1$ $\quad 9^{k+1} - 3^{k+1}$

$9 \cdot 9^k - 3 \cdot 3^k$

$(6+3)\, 9^k - 3 \cdot 3^k$ 3 has appeared so $9 = 6+3$

$6(9^k) + 3(9^k) - 3(3^k)$

$6(9^k) + 3(9^k - 3^k)$ same above

Prove $2^{3n-1} + 3$ is divisible by 7.

$n = 1$ $\quad 2^2 + 3 = 7$

$n = k$ $\quad 2^{3k-1} + 3$

$n = k+1$ $\quad 2^{3(k+1)-1} + 3$

$2^{3k+3-1} + 3$ get back to assumptn

$2^3 \cdot 2^{3k-1} + 3$

$8 \cdot 2^{3k-1} + 3$

$(7+1)\, 2^{3k-1} + 3$

$7 \cdot 2^{3k-1} + 2^{3k-1} + 3$ same as above.

Prove $n^2 + 3n + 2$ is divisible by 2.

$n = 1$ $\quad 1 + 3 + 2 = 6 \div 2 = 3$

$n = k$ $\quad k^2 + 3k + 2$ is divisible by 2.

$n = k+1$ $\quad (k+1)^2 + 3(k+1) + 2$ got to get back to assumptn

$k^2 + 2k + 1 + 3k + 3 + 2$

$k^2 + 3k + 2 + 2k + 4$

$k^2 + 3k + 2$ true from assumptn

$2k + 4 = 2(k+2)$ true because of 2