

Proof by Contradiction.

To prove a statement is true
 prove the opposite is false.

Note

Even number = $2n$, $n \in \mathbb{N}$
 = Can be divided by 2.
 = 2 is a factor.

Prove $\sqrt{2}$ is irrational.

Irrational \Rightarrow cannot be written as a fraction.

$\sqrt{2}$ is rational

$\Rightarrow \sqrt{2} = \frac{a}{b}$ where a and b have no common factor.

$$2 = \frac{a^2}{b^2}$$

$$2b^2 = a^2$$

$2b^2$ is even $\Rightarrow a^2$ must be even.

$\Rightarrow a$ must be even

$\Rightarrow a = 2k$

$$2b^2 = (2k)^2$$

$$2b^2 = 4k^2$$

$$b^2 = 2k^2$$

$\Rightarrow b^2$ must be even

$\Rightarrow b$ must be even

a and b are both so a common factor of 2. This is a contradiction to statement above

$\sqrt{2}$ is not rational \Rightarrow irrational.