De Moivre's Theorem

$z = (\cos\theta + i\sin\theta)$ use proof by induction to prove

$$z^n = (\cos n\theta + i\sin n\theta)$$

Prove $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

Prove for $n = 1$

$$\cos\theta + i\sin\theta = \cos\theta + i\sin\theta$$

Assume for. $n = k$

$$(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$$

Prove $n = k+1$

$$(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + \sin(k+1)\theta$$

$$(\cos\theta + i\sin\theta)^k(\cos\theta + i\sin\theta)$$

$$(\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$$

$$\cos k\theta\cos\theta + i\cos k\theta\sin\theta + i\sin k\theta\cos\theta + i^2\sin k\theta\sin\theta$$

$$\cos k\theta\cos\theta - \sin k\theta\sin\theta + i(\cos k\theta\sin\theta + \sin k\theta\cos\theta)$$

$$\cos(k\theta + \theta) + i\sin(k\theta + \theta)$$

$$\cos(k+1)\theta + i\sin(k+1)\theta$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

Conclusion no matter what is written down

Since true for $n = 1$ and $n = k+1$ then true for all n.