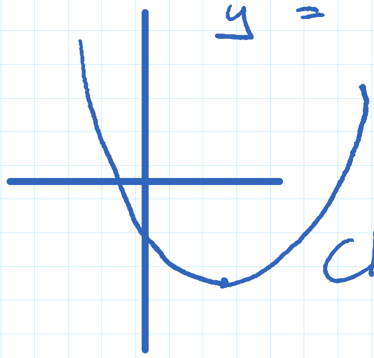
Turning points = stationary points = local maximum and local minimum points.

Find turning point of $y = x^2 - 6x + 1$ and sketch $y = x^2 - 6x + 1$.

$\frac{dy}{dx} = 0$ $\quad$ $\frac{dy}{dx} = 2x - 6 = 0$

$x = 3$

$y = 3^2 - 6(3) + 1 = -8$



$(3, -8)$

Slope $-$ 0 $+$

Change in slope

$\frac{d^2y}{dx^2} > 0$

$y = f(x) = $ function.

$\frac{dy}{dx} = f'(x) = $ slope

$\frac{d^2y}{dx^2} = f''(x) = $ slope of slope

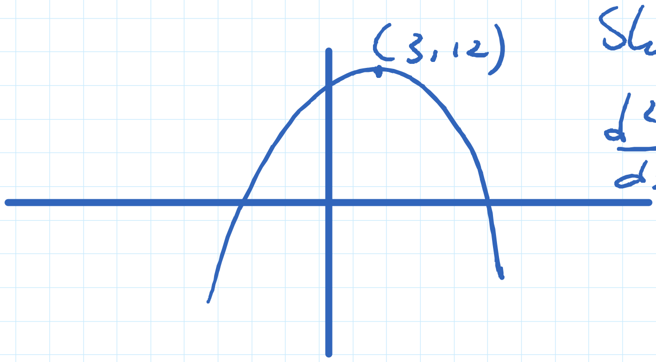
Find turning point of $y = 3 + 6x - x^2$.
Hence sketch $y = 3 + 6x - x^2$.

$y = 3 + 6x - x^2$

$\frac{dy}{dx} = 6 - 2x = 0$

$x = 3$

$y = 3 + 6(3) - 3^2 = 12$



Slope $+ \ 0 \ -$

$\frac{d^2y}{dx^2} < 0$

Maximum $\boxed{\frac{dy}{dx} = 0}$ and $\frac{d^2y}{dx^2} < 0$

Minimum $\boxed{\frac{dy}{dx} = 0}$ and $\frac{d^2y}{dx^2} > 0$

Find turning points of $y = x^3 - 3x^2 + 1$. State their nature.

$\frac{dy}{dx} = 3x^2 - 6x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \qquad x = 2$

$y = x^3 - 3x^2 + 1$

$x = 0 \quad y = 1 \quad (0,1)$

$x = 2 \quad y = -3 \quad (2,-3)$

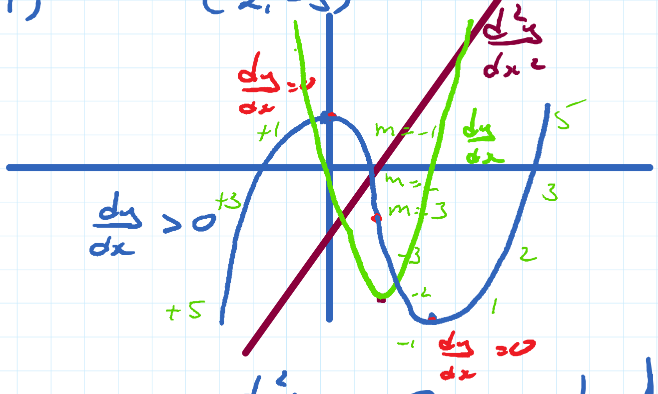
$\frac{d}{dx}y \qquad \frac{dy}{dx} = 3x^2 - 6x$

$\frac{d}{dx}\frac{dy}{dx} \qquad \frac{d^2y}{dx^2} = 6x - 6$

$x = 0 \qquad \frac{d^2y}{dx^2} = -6 < 0 \Rightarrow$ maximum

$x = 2 \qquad \frac{d^2y}{dx^2} = 6 > 0 \Rightarrow$ minimum

$(0,1) \qquad (2,-3)$



$\frac{d^2y}{dx^2} = 0$ = pt of inflection.

$f(x) = x^3 - 3x^2 - 9x + 1.$ Find turnings and point of inflection. Sketch a graph.

$$f(x) = x^3 - 3x^2 - 9x + 1$$

$$f'(x) = 3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \qquad x = -1$$

$$f(3) = 3^3 - 3(3)^2 - 9(3) + 1 = -26$$

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 1 = 6$$

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$f''(3) = 12 > 0 \Rightarrow$ minimum

$f''(-1) = -12 < 0 \Rightarrow$ maximum

$6x - 6 = 0$

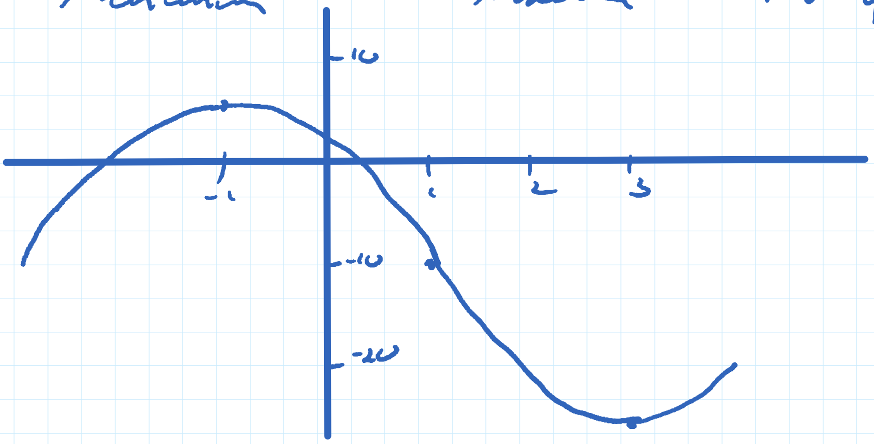
$x = 1$

$f(1) = 1^3 - 3(1) - 9(1) + 1 = -10$

$(3, -26)$ Minimum

$(-1, 6)$ Maximum

$(1, -10)$ Pt of inflect.



Note : Pt of inflection is midpoint of maximum and minimum point.

$$\boxed{\text{Point of inflection } \frac{d^2y}{dx^2} = 0}$$

Find turning points of

$f(x) = \dfrac{3x}{x-5}$ where $x \in R$, $x \neq 5$

$u = 3x$ $\qquad v = x - 5$

$\dfrac{du}{dx} = 3$ $\qquad \dfrac{dv}{dx} = 1$

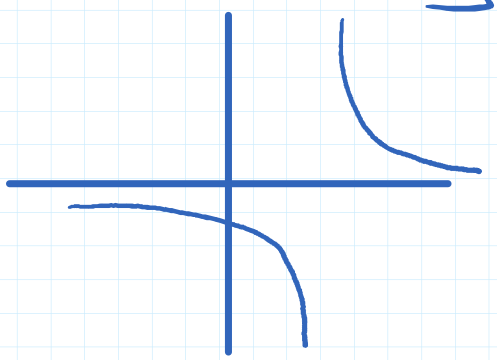
$$\frac{dy}{dx} = \frac{3(x-5) - 3x}{(x-5)^2}$$

$$= \frac{\cancel{3x} - 15 - \cancel{3x}}{(x-5)^2} = 0$$

$$\Rightarrow \quad -15 = 0$$

impossible $\Rightarrow$ no turning points

It is a decreasing curve.



Find turning point of

$y = x \ln x$

$$y = x \ln x$$

$$\frac{dy}{dx} = x \frac{1}{x} + \ln x = 0$$

$$1 + \ln x = 0$$

$$\ln x = -1$$

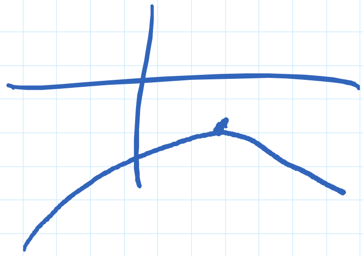
$$\log_e x = -1$$

$\ln e^x = x$

$$e^{-1} = x = \frac{1}{e}$$

$$y = e^{-1} \ln e^{-1} = \frac{-1}{e}$$

$\left(\frac{1}{e}, -\frac{1}{e}\right)$



The function $f(x) = ax^3 + bx^2 + cx + d$ has a maximum point at (0, 4) and a point of inflection at (1, 0).

Find the values of a, b, c and d.

$$f'(x) = 3ax^2 + 2bx + c = 0$$

$$f'(0) = c = 0$$

$$f''(x) = 6ax + 2b = 0$$

$$f''(1) = 6a + 2b = 0$$

$$3a + b = 0$$

$$y = ax^3 + bx^2 + cx + d \qquad (0, 4)$$

$$x \quad y$$

$$4 = d$$

$$(1, 0)$$

$$0 = a + b + c + d$$

$$c = 0 \qquad d = 4$$

$$a + b = -4$$

$$3a + b = 0$$

$$-2a = -4$$

$$a = 2 \qquad b = -6$$

Let $g(x) = x^2 + \dfrac{a}{x^2}$ where a is a real number and $x \in \mathbf{R}$, $x \neq 0$.

Given that $g(x)$ has a turning point at $x = 2$,

(i) find the value of a

(ii) prove that $g(x)$ has no local maximum points.

$$y = x^2 + ax^{-2}$$

$$\frac{dy}{dx} = 2x - 2ax^{-3} = 0$$

$$2x - \frac{2a}{x^3} = 0$$

$x = 2$ $\quad 2(2) - \dfrac{2a}{2^3} = 0$

$$32 = 2a \Rightarrow a = 16$$

$a = 16$ $\quad 2x - \dfrac{32}{x^3} = 0$

$$2x^4 = 32 \qquad x^4 = 16$$

$$x = \pm 2.$$

$$\frac{dy}{dx} = 2x - 32x^{-3}$$

$$\frac{d^2y}{dx^2} = 2 + 96x^{-4} = 2 + \frac{96}{x^4} > 0$$

$x^4 = (x^2)^2 > 0$ Any number squared is positive.