Identities.

Prove $\cos A \tan A = \sin A$

Rules (i) Each side is separate.
(ii) Change to $\sin A$ and $\cos A$
(iii) Use tables or algebra.

$$\cos A \tan A = \sin A$$

$$\cancel{\cos A} \cdot \frac{\sin A}{\cancel{\cos A}} = \sin A$$

QED

Prove $\sin A . \tan A + \cos A = \sec A.$

$$\sin A . \frac{\sin A}{\cos A} + \cos A = \frac{1}{\cos A}$$

$$\frac{\sin^2 A}{\cos A} + \frac{\cos A}{1}$$

$$\frac{\sin^2 A + \cos^2 A}{\cos A}$$

$$\frac{1}{\cos A} \quad \text{QED}$$

Prove

$$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \tan\theta$$

$$\frac{\sin\theta}{\sqrt{\cos^2\theta}} = \frac{\sin\theta}{\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta}$$

$$\boxed{\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \sin^2\theta &= 1 - \cos^2\theta \\ \cos^2\theta &= 1 - \sin^2\theta \end{aligned}}$$

$$\frac{(1+\cos\theta)(1-\cos\theta)}{\cos^2\theta} = \tan^2\theta$$

$$\frac{1-\cos^2\theta}{\cos^2\theta} = \quad = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\frac{\sin^2\theta}{\cos^2\theta}$$

QED

$$(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = 2$$

$$\cos^2\theta + \cancel{2\cos\theta\sin\theta} + \sin^2\theta + \cos^2\theta - \cancel{2\cos\theta\sin\theta} + \sin^2\theta$$

$$2(\cos^2\theta + \sin^2\theta)$$

$$2(1) = 2$$

QED

$$\frac{1}{1-\sin A} + \frac{1}{1+\sin A} = 2\sec^2 A.$$

$$\frac{1+\cancel{\sin A} + 1-\cancel{\sin A}}{(1-\sin A)(1+\sin A)} = 2\left(\frac{1}{\cos^2 A}\right)$$

$$\frac{2}{1-\sin^2 A} = \frac{2}{\cos^2 A}$$

$$\frac{2}{\cos^2 A} = \frac{2}{\cos^2 A}$$

$$\frac{\tan\theta + \sin\theta}{\sec\theta + 1} = \sin\theta$$

$$\frac{\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{1}}{\frac{1}{\cos\theta} + \frac{1}{1}} = \sin\theta$$

$$\frac{\frac{\sin\theta + \sin\theta\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cancel{\cos\theta}}}$$

$$\frac{\sin\theta + \sin\theta\cos\theta}{1+\cos\theta}$$

$$\frac{\sin\theta(\cancel{1+\cos\theta})}{\cancel{1+\cos\theta}}$$

Double and Compound angle.

Prove

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

$$2\sin A\cos A = \frac{2\frac{\sin A}{\cos A}}{1+\frac{\sin^2 A}{\cos^2 A}}$$

$$\frac{1}{1} + \frac{\sin^2 A}{\cos^2 A}$$

$$\frac{\cos^2 A + \sin^2 A}{\cos^2 A}$$

$$\frac{1}{\cos^2 A}$$

$$= \frac{\frac{2\sin A}{\cos A}}{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}$$

$$= \frac{\frac{2\sin A}{\cos A}}{\frac{1}{\cos^2 A}}$$

$$= \frac{2\sin A}{\cancel{\cos A}} \cdot \frac{\cos^2 A}{1}$$

$$= 2\sin A\cos A \quad \underline{\text{QED}}$$

$$1 - (\cos x - \sin x)^2 = \sin 2x$$

$$1 - (\cos^2 x + \sin^2 x - 2\sin x\cos x = 2\sin x\cos x$$

$$\cancel{1-1} + 2\sin A\cos A$$

$$\cancel{\frac{\sin^2\theta}{\cos^2\theta} \quad \cos^2\theta}$$

Prove

$$\frac{\cos 2A}{\cos A + \sin A} = \cos A - \sin A.$$

$a^2 - b^2$

$$\frac{\cos^2 A - \sin^2 A}{\cos A + \sin A} = \cos A - \sin A$$

$$\frac{(\cos A - \sin A)\cancel{(\cos A + \sin A)}}{\cancel{\cos A + \sin A}}$$

Show

$$\frac{\sin 2A}{1 + \cos 2A} = \tan A.$$

Hence prove $\tan 22\tfrac{1}{2} = \sqrt{2} - 1$

$$\frac{2\sin A\cos A}{1 + \cos^2 A - \sin^2 A} \qquad \frac{\sin A}{\cos A}$$

$\cos^2 A + \sin^2 A = 1$

$$\frac{\cancel{2}\sin \bar{A}\,\cancel{\cos A}}{\cancel{2}\cos^2 A}$$

$1 - \sin^2 A + \cos^2 A$

$\cos^2 A + \cos^2 A$

$2\cos^2 A.$

$$\tan A = \frac{\sin 2A}{1 + \cos 2A} \qquad \text{Let } A = 22\tfrac{1}{2}$$

$$\tan 22\tfrac{1}{2} = \frac{\sin 45}{1 + \cos 45}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{1} + \frac{1}{\sqrt{2}}}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}} = \frac{1}{\cancel{\sqrt{2}}} \cdot \frac{\cancel{\sqrt{2}}}{1+\sqrt{2}}$$

$$(1+\sqrt{2})(1-\sqrt{2})$$

$$1-\sqrt{2}^2$$

$$1-2 = 1$$

$$= \frac{1}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}}{-1}$$

$$= \sqrt{2}-1$$

Small Letters

Prove

$$bc\cos A + ac\cos B = c^2$$

$$a^2 = b^2+c^2-2bc\cos A$$

$$2bc\cos A = b^2+c^2-a^2$$

$$bc\cos A = \frac{b^2+c^2-a^2}{2}$$

$$b^2 = a^2+c^2-2ac\cos B$$

$$2ac\cos B = a^2+c^2-b^2$$

$$ac\cos B = \frac{a^2+c^2-b^2}{2}$$

$$bc\cos A + ac\cos B$$

$$\frac{\cancel{b^2}+c^2-\cancel{a^2}}{2} + \frac{\cancel{a^2}+c^2-\cancel{b^2}}{2}$$

$$\frac{2c^2}{2} = c^2$$

Prove $\quad b\cos C + c\cos B = a$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$2ab\cos C = a^2 + b^2 - c^2$$

$$b\cos C = \frac{a^2 + b^2 - c^2}{2a}$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$2ac\cos B = a^2 + c^2 - b^2$$

$$c\cos B = \frac{a^2 + c^2 - b^2}{2a}$$

$$\frac{a^2 + \cancel{b^2} - \cancel{c^2}}{2a} + \frac{a^2 + \cancel{c^2} - \cancel{b^2}}{2a}$$

$$\frac{2a^2}{2a} = a$$