Solve $x^2 - 2x - 8 = 0$

$f(x) = x^2 - 2x - 8$

$f(1) = 1 - 2 - 8 \neq 0$

$f(2) = 2^2 - 2(2) - 8 \neq 0$

$x^2 - 2x - 8 = 0$ C/V -8

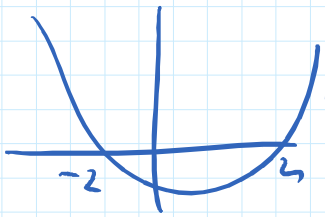
$x^2 - 4x + 2x - 8 = 0$ Sub -2

$x(x-4) + 2(x-4) = 0$

$(x-4)(x+2) = 0$ Factors

$x - 4 = 0$ $\quad x + 2 = 0$

$x = 4$ $\quad x = -2$



$f(4) = 4^2 - 2(4) - 8 = 0$

$f(-2) = (-2)^2 - 2(-2) - 8 = 0$

$f(x) = ax^3 + bx^2 + cx + d$ given $f(k) = 0$

$\Rightarrow x = k$ is a root

$\Rightarrow x - k$ is a factor.

Find factors of $x^3 - 8x^2 + 19x - 12$

$f(x) = x^3 - 8x^2 + 19x - 12$

$f(1) = 1 - 8 + 19 - 12 = 0$

$x = 1$ is a root

$x - 1$ is a factor

$$x-1 \overline{)\, x^3 - 8x^2 + 19x - 12}$$

$x^2 - 7x + 12$

$-x^3 + 1x^2$

$-7x^2 + 19x$

$\pm 7x^2 \mp 7x$

$12x - 12$

$12x - 12$

0

$$11 \overline{)\, 1573}$$

143

-11

47

-44

33

33

0

$x^2 - 7x + 12$

$(x-3)(x-4)$

Ans $(x-1)(x-3)(x-4)$

Find factors of $x^3 + 6x^2 - x - 30$

$f(2) = 2^3 + 6(2)^2 - 2 - 30$

$8 + 24 - 2 - 30 = 0$

$x = 2$ root

$x - 2$ is a factor

$$x-2 \overline{)\, x^3 + 6x^2 - x - 30}$$

$x^2 + 8x + 15$

$-x^3 \pm 2x^2$

$8x^2 - x$

$-8x^2 + 16x$

$15x - 30$

$15x - 30$

0

$x^2 + 8x + 15$

$(x+3)(x+5)$

Note $(-1)^n = +1 \Rightarrow n$ is even

$(-1)^n = -1 \Rightarrow n$ is odd

$(-1)^3 = (-1)(-1)(-1) = -1$

Even $= 2n$

Odd $= 2n+1$

$(-1)^{2n} = +1$

$(-1)^{2n+1} = -1$, $n \in \mathbb{Z}$

Solve $x^3 - 6x^2 + 32 = 0$ hence sketch $y = x^3 - 6x^2 + 32$.

$f(x) = x^3 - 6x^2 + 32$

$f(-2) = (-2)^3 - 6(-2)^2 + 32 = 0$

$x = -2$ is a root $\Rightarrow x + 2$ is a factor

$$x+2 \overline{)\, x^3 - 6x^2 + 0x + 32} \quad = \quad x^2 - 8x + 16$$

Note $0x$

$-x^3 \mp 2x^2$

$-8x^2 + 0x$

$-8x^2 + 16x$

$+16x + 32$

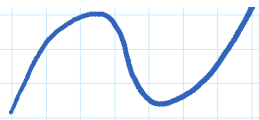
$16x + 32$

0

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0$

$x = 4 \quad x = 4$

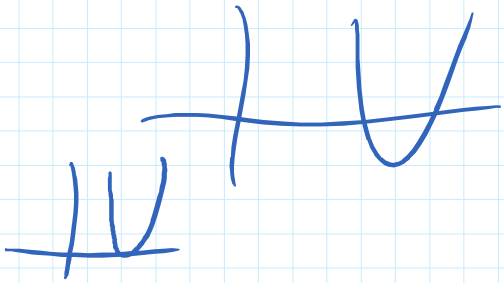
x^3 

$x = -2$ $x = 4$ $x = 4$

If Quad

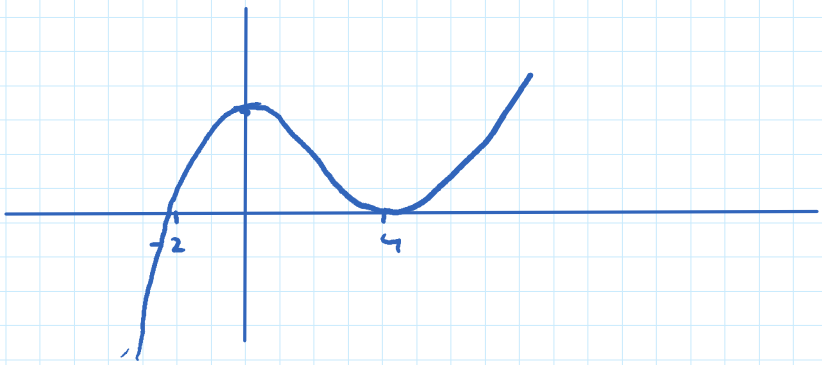
If Quad $x = 1$ $x = 5$

$x = 2$ $x = 2$



$y = x^3 - 6x^2 + 32$

$x = 0$ $y = 32$ $(0, 32)$



Solve $x^3 - 8x^2 + 19x - 12 = 0$ and hence sketch $y = x^3 - 8x^2 + 19x - 12$

$f(x) = x^3 - 8x^2 + 19x - 12$

$f(1) = 1 - 8 + 19 - 12 = 0$

$x = 1$ is a root

$x - 1$ is a factor

$$x-1 \overline{)\, x^3 - 8x^2 + 19x - 12}$$

$x^2 - 7x + 12$

$-x^3 \pm 1x^2$

$-7x^2 + 19x$

$\pm 7x^2 \mp 7x$

$12x - 12$

$12x - 12$

0

$$(x-1)(x^2 + ax + b) = x^3 - 8x^2 + 19x - 12$$

$$x^3 + ax^2 + bx - x^2 - ax - b = x^3 - 8x^2 + 19x - 12$$

$a - 1 = 8$ $\quad -b = -12$

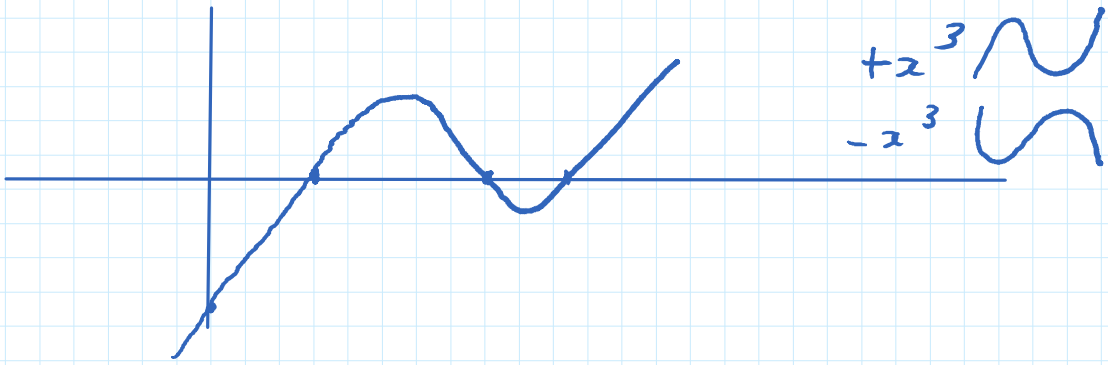
$a = 7$ $\quad b = 12$

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$

$x = 3$ $\quad x = 4$

$x = 0$ $\quad y = -12$



$x+2$ is a factor of x^3+6x^2+px+6

find p.

$$x+2=0$$
$$x=-2$$
$$(-2)^3+6(-2)^2+p(-2)+6=0$$
$$-8+24-2p+6=0$$
$$-2p=-22$$
$$p=11$$

$$x+2 \;\overline{)\; x^3+6x^2+px+6}$$
$$x^2+4x+p-8$$
$$-x^3-2x^2$$
$$4x^2+px$$
$$-4x^2-8x$$
$$(p-8)x+6$$
$$-(p-8)x-2(p-8)$$
$$6-2(p-8)=0$$
$$6-2p+16=0$$
$$p=11$$

- $f(x)=3x^3+mx^2-17x+n$, where m and n are constants.
Given that $x-3$ and $x+2$ are factors of $f(x)$, find the value of m and the value of n.

$$f(x)=3x^3+mx^2-17x+n$$
$$f(3)=3(27)+9m-51+n=0$$
$$9m+n=-30$$
$$f(-2)=3(-2)^3+m(-2)^2-17(-2)+n=0$$
$$-24+4m+34+n=0$$

$4m + n = -10$

$$\begin{array}{r} 9m + n = -30 \\ -4m + -n = \pm 10 \\ \hline 5m = -20 \end{array}$$

$m = -4$

$9(-4) + n = -30$

$n = 6$

Given $x^2 - ax - 3$ is a factor of $x - 5x^2 + bx + 9$ find a and b.

$(x + k)(x^2 - ax - 3) = x^3 - 5x^2 + bx + 9.$

$x^3 - ax^2 - 3x - kx^2 - akx - 3k = x^3 - 5x^2 + bx + 9$

$-a + k = -5$

$-3 - ak = b$

$-3k = 9$

$k = -3$

$-a - 3 = -5$

$-a = -2$

$a = 2$

$-3 - 2(-3) = b$

$b = 3$

$(x - a)^2$ is a factor of $x^3 + 3px + q$. Show

(i) $p = -a^2$ (ii) $q = 2a^3$

$$(x-a)^2 = x^2 - 2ax + a^2$$

$$(x+k)(x^2-2ax+a^2) = x^3 + 0x^2 + 3px + q$$

$$x^3 - 2ax^2 + a^2x + kx^2 - 2akx + ka^2 = x^3 + 0x^2 + 3px + q$$

$$-2a + k = 0$$

$$k = 2a$$

$$a^2 - 2ak = 3p$$

$$a^2 - 2a(2a) = 3p$$

$$a^2 - 4a^2 = 3p$$

$$-3a^2 = 3p$$

$$p = -a^2$$

$$ka^2 = q$$

$$2a(a^2) = q$$

$$q = 2a^3$$