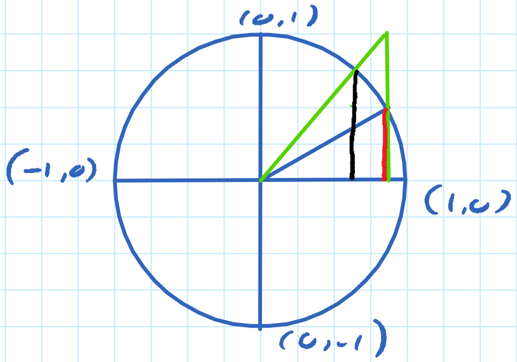
Find $\sin 30 = 0.5$

$\sin 60 = 0.866$

$$\sin 30 = \frac{1}{2}$$



$\sin 2A$, $\cos 2A$, $\tan 2A$

$$\sin 2A \neq 2\sin A$$

$$\left.\begin{aligned} \sin 2A &= 2\sin A\cos A \\ \sin 2A &= \frac{2\tan A}{1+\tan^2 A} \end{aligned}\right\} \text{Tables.}$$

Type 1.

$\sin A = \frac{3}{5}$ find $\sin 2A$.

$$\sin 2A \neq 2\left(\frac{3}{5}\right) = \frac{6}{5}$$

$$\sin 2A = 2\sin A\cos A$$

$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$

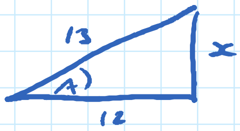
$$\sin 2A = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

$\cos A = \frac{12}{13}$ find $\cos 2A$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos^2 A = (\cos A)^2$$

$$\cos^2 A \neq \cos A^2$$



$x^2 + 12^2 = 13^2$

$x = 5$

$\cos 2A = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{119}{169}$

Type 2:

$\sin 2A = \frac{4}{5}$ find 2 values for $\sin A$.

$\sin 2A = 2\cos A \sin A = \frac{4}{5}$

$\sin 2A = \frac{2\tan A}{1+\tan^2 A} = \frac{4}{5}$

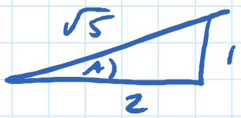
$10t = 4(1+t^2) \qquad t = \tan A$

$10t = 4 + 4t^2$

$2t^2 - 5t + 2 = 0$

$(2t-1)(t-2) = 0$

$\tan A = \frac{1}{2} \qquad \tan A = \frac{2}{1}$



$\sin A = \frac{1}{\sqrt{5}} \qquad \sin A = \frac{2}{\sqrt{5}}$

Compound Angles.

Compound = 2 angles

Foirmlí uillinneacha comhshuite — **Compound angle formulae**

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

A (céimeanna)	0°	90°	180°	270°	30°	45°	60°	A (degrees)
A (raidiain)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	A (radians)
$\cos A$	1	0	–1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\cos A$
$\sin A$	0	1	0	–1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\sin A$
$\tan A$	0	-	0	-	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\tan A$

Type 1

Find

$$\sin 15^\circ = \sin(60-45)$$

$$\sin 60 \cos 45 - \cos 60 \sin 45$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$\frac{\sqrt{3}-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos 75 = \cos(45+30)$$

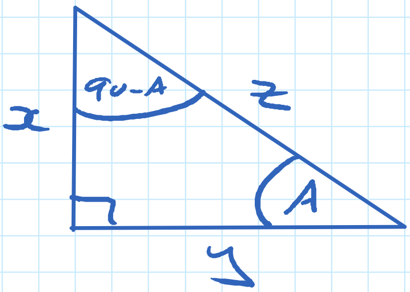
$$= \cos 45 \cos 30 - \sin 45 \sin 30$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\sin 15 = \cos 75$$

$$\sin 30 = \cos 60$$

$$\sin A = \cos(90-A)$$



$$\sin A = \frac{x}{z}$$

$$\cos A = \frac{y}{z}$$

$$\sin(90-A) = \frac{y}{z}$$

$$\cos(90-A) = \frac{x}{z}$$

$$\sin(90-A) = \cos A$$

$$\cos(90-A) = \sin A$$

$$\tan 105^\circ$$

$$\tan 105 = \tan(45+60)$$

$$= \frac{\tan 45 + \tan 60}{1 - \tan 45 \tan 60}$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

Top $\quad 1+\sqrt{3}+\sqrt{3}+3 = 4+2\sqrt{3}$

Bottom $\quad (1-\sqrt{3})(1+\sqrt{3}) = 1-3 = -2$

$\tan 105 = -2-\sqrt{3}$

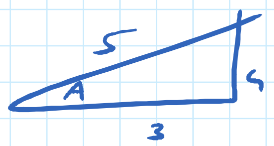
Type 2:

$\cos A = \frac{3}{5}$, $\sin B = \frac{5}{13}$ find

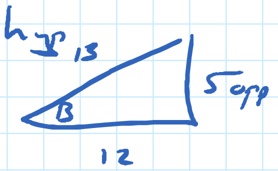
(i) $\cos(A+B)$

(ii) $\cos(45+A+B)$.

(i) $\cos(A+B) = \cos A\cos B - \sin A \sin B$



$\sin A = \frac{4}{5}$



$\sin B = \frac{5}{13}$

$\cos B = \frac{12}{13}$

$$= \frac{3}{5}\left(\frac{12}{13}\right) - \frac{4}{5}\left(\frac{5}{13}\right)$$

$$= \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$$

(ii) $\cos(45+A+B)$

$$\neq \cos 45 + \cos(A+B)$$

$$\cos(A+B) \neq \cos A + \cos B$$

$$\cos(\underbrace{45}_{A} + \underbrace{A+B}_{B})$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$= \cos 45\cos(A+B) - \sin 45\sin(A+B)$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\frac{4}{5}\left(\frac{12}{13}\right) + \frac{3}{5}\left(\frac{5}{13}\right) = \frac{63}{65}$$

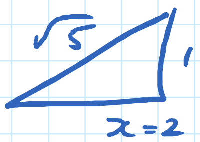
$$\cos(45 + A + B) = \frac{1}{\sqrt{2}} \cdot \frac{16}{65} - \frac{1}{\sqrt{2}} \cdot \frac{63}{65}$$

$$= -\frac{47}{65\sqrt{2}}$$

$\sin A = \frac{1}{\sqrt{5}}$ $\cos B = \frac{2}{\sqrt{7}}$ find $\sin(A+B)$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

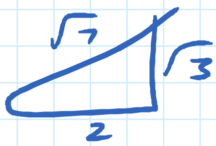
$\sin A = \frac{1}{\sqrt{5}}$



$x^2 + 1 = 5$
$x^2 = 4$
$x = 2$

$\cos A = \frac{2}{\sqrt{5}}$

$\cos B = \frac{2}{\sqrt{7}}$



$\sin B = \frac{\sqrt{3}}{\sqrt{7}}$

$$\sin(A+B) = \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{7}} + \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{3}}{\sqrt{7}}$$

$$= \frac{2 + 2\sqrt{3}}{\sqrt{35}}$$

Type 3:

$\tan A = 5$ $\tan(A+B) = 7$ find $\tan B$.

$$\tan(A+B) = 7$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\boxed{\tan(A+B) \neq \tan A + \tan B}$$

$$7 = \frac{5 + t}{1 - 5t} \qquad t = \tan B$$

$$7(1 - 5t) = 5 + t$$

$$7 - 35t = 5 + t$$

$$2 = 36t$$

$\tan B = \frac{1}{18}$

Type 4:

Prove $\cos 3A = 4\cos^3 A - 3\cos A.$

Bigger down

$$\cos 3A = \cos(2A + A)$$

$$\cos(A+B) = \cos A\cos B - \sin A \sin B$$

$$= \cos 2A \cos A - \sin 2A \sin A$$

$$= (\cos^2 A - \sin^2 A)\cos A - 2\sin A\cos A \sin A$$

$$= \cos^3 A - \sin^2 A\cos A - 2\sin^2 A\cos A$$

$$= \cos^3 A - 3\sin^2 A\cos A$$

$$= \cos^3 A - 3\cos A(1 - \cos^2 A)$$

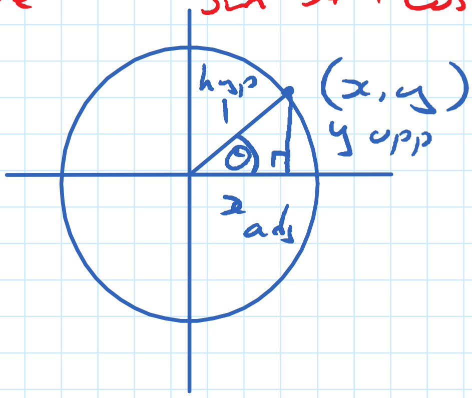
$$= \cos^3 A - 3\cos A + 3\cos^3 A$$

$$= 4\cos^3 A - 3\cos A.$$

$$\boxed{\sin^2 A + \cos^2 A = 1}$$

$$\sin^2 A = 1 - \cos^2 A$$

Prove $\sin^2 A + \cos^2 A = 1$



$\sin A = y$

$\cos A = x$

$x^2 + y^2 = 1$

$(\cos A)^2 + (\sin A)^2 = 1$

$\cos^2 A + \sin^2 A = 1$