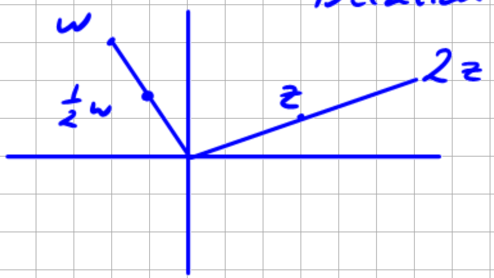
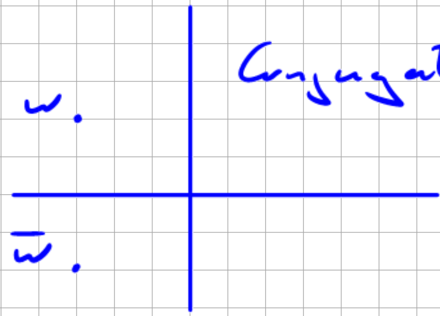
Diagrams.

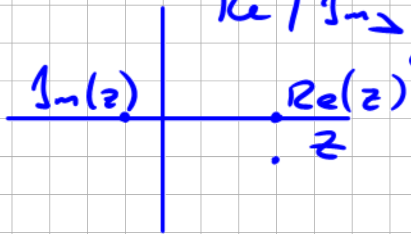
Dilation = same line



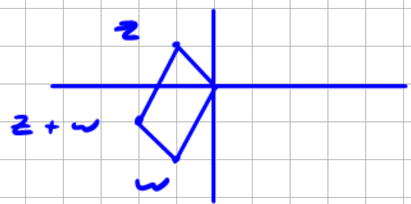
Conjugate = above each other



Re / Img both on real axis



Add = parallelogram
z + w

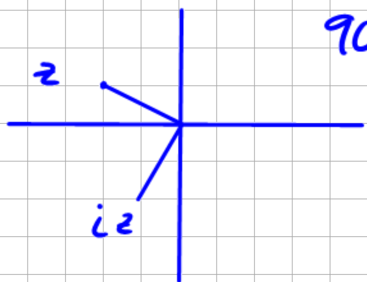


$z = x + yi$

$x = \text{Re } z$

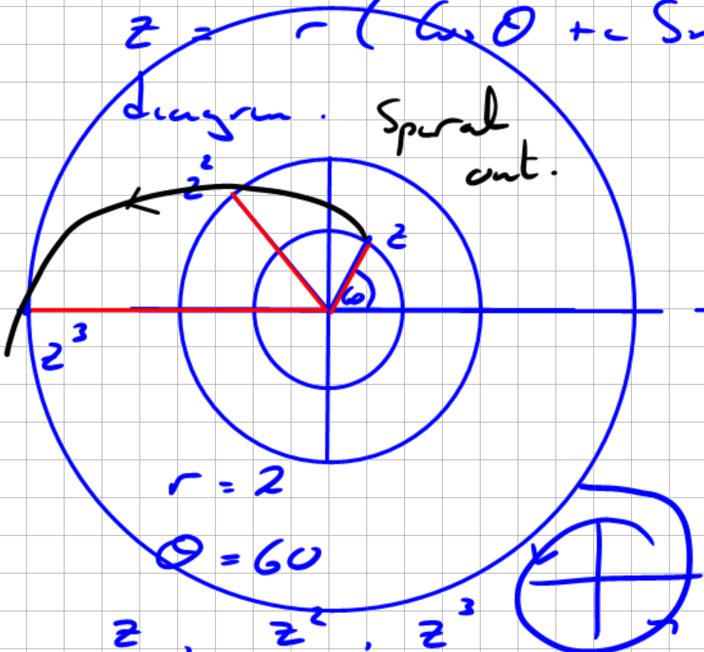
$y = \text{Img } z$

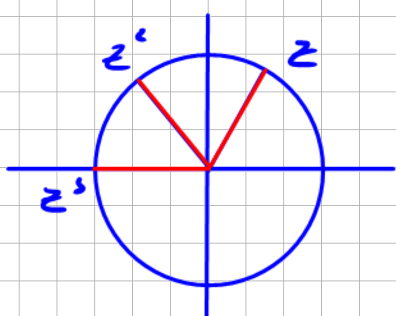
90° anti clockwise rotation same distance from origin.

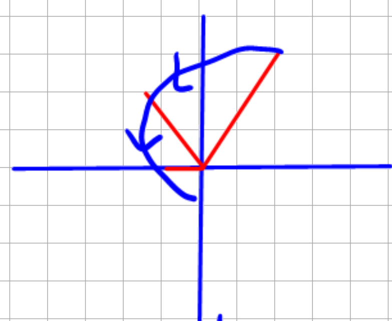


90° anti clockwise rotation.

$z = r(\cos\theta + i\sin\theta)$ plot on Argand diagram. Spiral out.



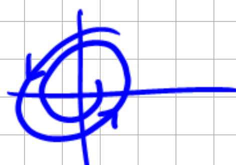




$r = 2$ $\quad r = 1$ $\quad r = \frac{1}{2}$

$\theta = 60$ $\quad \theta = 60°$ $\quad \theta = 60°$

z, z^2, z^3



Measure angles with protractor.
Lengths with a compass.

$z^2 = 4(\cos 120 + i\sin 120)$
$z^3 = 8(\cos 180 + i\sin 180)$

2014 Sample Paper Q1 (b)

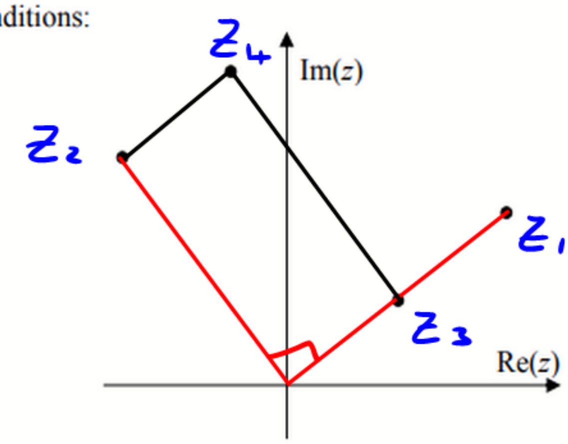
Four complex numbers z_1, z_2, z_3 and z_4 are shown on the Argand diagram. They satisfy the following conditions:

$z_2 = iz_1$

$z_3 = kz_1$, where $k \in \mathbb{R}$

$z_4 = z_2 + z_3$.

The same scale is used on both axes.



(i) Identify which number is which, by labelling the points on the diagram.

(ii) Write down the approximate value of k.

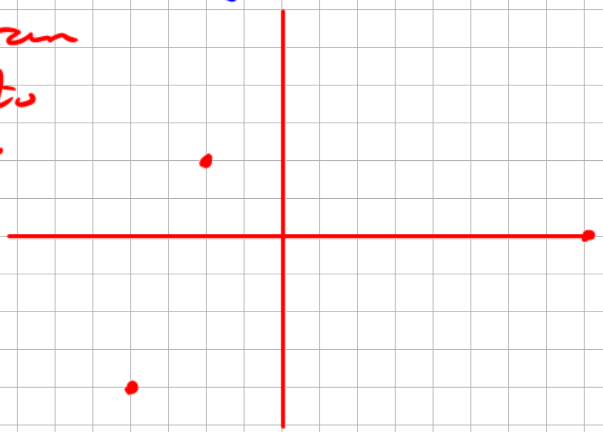
Answer: $\frac{1}{2}$

$z_3 = kz_1$ $\Rightarrow$ same line to origin.

90° anti clockwise rotation = same distance from origin.

z_4 = parallelogram.

Diagram not to scale.



$|r| > 1$

Diagram shows z, z^2 and z^3.

Find θ and given $r = 2$ find z, z^2 and z^3 in rectangular form.

$$z = r(\cos\theta + i\sin\theta)$$

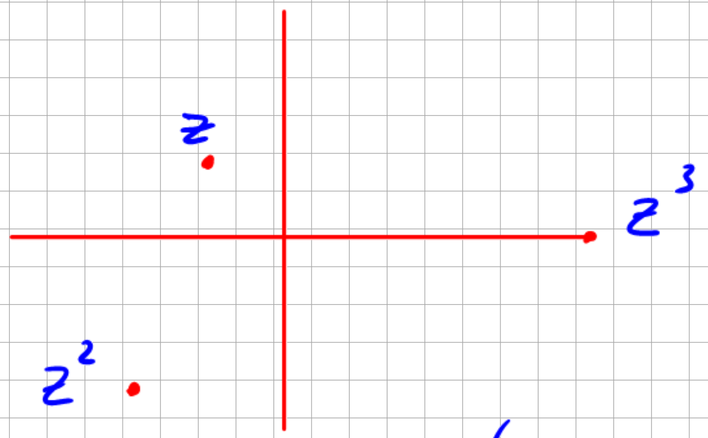
$r = |z|$ = distance from origin.

$$z^2 = (r(\cos\theta + i\sin\theta))^2$$
$$= r^2(\cos 2\theta + i\sin 2\theta)$$
$$z^3 = r^3(\cos 3\theta + i\sin 3\theta)$$

$z \rightarrow z^2 \rightarrow z^3$

$r \rightarrow r^2 \rightarrow r^3 \Rightarrow$ further away from origin

$\theta,\ 2\theta,\ 3\theta \Rightarrow$ rotating anti-clockwise



$r = 2$

$z^3 = r^3(\cos 3\theta + i\sin 3\theta)$

Diagram z^3 is $360°$

$3\theta = 360$

$\theta = 120° =$ argument

$z = 2(\cos 120 + i\sin 120)$

$= -1 + \sqrt{3}i$

$z^2 = 4(\cos 240 + i\sin 240)$

$-2 - 2\sqrt{3}i$

$z^3 = 8(\cos 360 + i\sin 360)$

$8 + 0i$

rectangular form is $a + bi$.