Loans.

A car cost €6,500. I borrow the whole amount at APR of 8.6%. It is paid back at end of month in equal instalments for 3 years. What are the value of instalments?

$$(1+i)^{12} = 1.086$$

$$1+i = \sqrt[12]{1.086}$$

$$= 1.007$$

$P = 6500$ $F = ?$

$$6500 = \frac{F}{1.007} + \frac{F}{(1.007)^2}$$

$$= F\left[\frac{1}{1.007} + \frac{1}{1.007^2} + \dots\right]$$

$a = \frac{1}{1.007}$ $r = \frac{1}{1.007}$ $n = 36$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{\frac{1}{1.007}\left(1 - \left(\frac{1}{1.007}\right)^{36}\right)}{1 - \frac{1}{1.007}}$$

$$6500 = 31.78F$$

$$€204.52 = F$$

I burrowed €x over 4 years at 9.6% APR. I paid back €250 each month at end of month. Find x.

$P = x \qquad F = 250$

$(1+i)^{12} = 1.096$

$1+i = \sqrt[12]{1.096} = 1.007$

$x = \frac{250}{1.007} + \frac{250}{1.007^2}$

$= 250\left[\frac{1}{1.007} + \frac{1}{1.007^2} + \dots\right]$

$a = \frac{1}{1.007} \qquad r = \frac{1}{1.007} \qquad n = 48$

$S_n = \frac{a(1-r^n)}{1-r} = \frac{\frac{1}{1.007}\left(1 - \frac{1}{1.007^{48}}\right)}{1 - \frac{1}{1.007}}$

$x = 40.03(250)$

$x = €10,007.61$

I borrow €5,000 over 3 years at 8% APR, paid back at end of each year in equal instalments. Find instalments and the schedule of payments.

$P = 5000 \qquad t = 3 \qquad i = 0.08 \qquad F =$

$$5000 = \frac{F}{1.08} + \frac{F}{(1.08)^2} + \frac{F}{(1.08)^3}$$

$$= F\left[\frac{1}{1.08} + \frac{1}{1.08^2} + \frac{1}{1.08^3}\right]$$

↑ Calculator

$$5000 = F(2.577)$$

$$F = \frac{5000}{2.577}$$

$$F = €1,940.17$$

Schedule of payment at end of year.

Time	Principal	Interest.
0	€ 5000	
1	€3459.83	€400
2	€ 1796.45	€ 276.79
3	€ 0	€ 143.72

Year 1 $\quad F = 5000(1.08) = 5400 - 1940.17 = 3459.83$

Year 2 $\quad F = 3459.83(1.08) = 3736.62 - 1940.17 = 1796.45$

Year 3 $\quad F = 1940.17(1.08) = €1940.17$

Amortisation Formula

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$A =$ annual repayments

$P =$ Principal.

€250,000 is borrowed over 30 years at 2.75% APR. Find monthly repayments.

$$(1+i)^{12} = 1.0275$$

$$= 1.002$$

$$i = 0.002 \qquad P = 250{,}000$$

$$t = 360$$

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$= 250{,}000\left(\frac{0.002(1.002)^{360}}{(1.002)^{360} - 1}\right)$$

$$= €1016.10$$

I borrow €x over 20 years at 2.75% APR paid in equal instalments at end of each month. If the instalments are €1250 find x.

$$(1+i)^{12} = 1.0275$$

$$1+i = \sqrt[12]{1.0275} = 1.002$$

$$A = 1250 \qquad P = P \qquad t = 20 \times 12 = 240$$

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$1250 = P\frac{0.002(1.002)^{240}}{(1.002)^{240} - 1}$$

$$1250 = P(0.0054)$$

$$231273.58 = P.$$

£P is borrowed over t years with annual repayment of A with APR of i%.
Prove

$$A = P\frac{i(1+i)^t}{(1+i)^t-1}$$

$$P = \frac{A}{(1+i)^t}$$

$$P = \frac{A}{1+i} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3}$$ Need this right

$$= A\left[\frac{1}{1+i} + \frac{1}{(1+i)^2} + \dots\right]$$ Series

$$a = \frac{1}{1+i} \qquad r = \frac{1}{1+i}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\left(\frac{1}{1+i}\right)^n = \frac{1^n}{(1+i)^2} = \frac{1}{(1+i)^n}$$

$$= \frac{\frac{1}{1+i}\left(\frac{1}{1} - \frac{1}{(1+i)^t}\right)}{\frac{1}{1} - \frac{1}{1+i}}$$ Algebra?

$$= \frac{\frac{1}{1+i}\left(\frac{(1+i)^t-1}{(1+i)^t}\right)}{\frac{\cancel{1}+i-\cancel{1}}{1+i}}$$ ← invert & mult.

$$= \frac{\cancel{1+i}}{i} \cdot \frac{1}{\cancel{1+i}}\left(\frac{(1+i)^t-1}{(1+i)^t}\right)$$

$$P = A \cdot \frac{(1+i)^t-1}{i(1+i)^t}$$

$$P.\ i(1+i)^t = A\left((1+i)^t-1\right)$$

$$P.\ \frac{i(1+i)^t}{(1+i)^t-1} = A$$