Solve $\quad 2x - 1 < 3 \quad , \quad x \in \mathbb{N}.$

$$2x < 4$$
$$x < 2$$
$$x = 1$$

$R | Q = $ irrational

$$2x - 1 < 3$$
$$2x < 4$$
$$x < 2$$

Solve $\quad x = 1$

$$1 - 3x > 10$$
$$-3x > 9$$
$$3x < -9$$
$$x < -3$$

$-3 < -1 \qquad 2 > 1$

Right $\quad > \quad$ Left
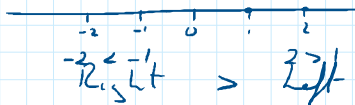
$$-3 \leq 2x - 1 < 5$$

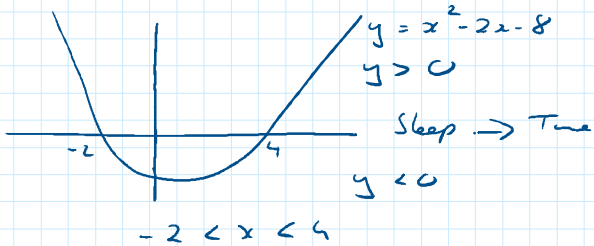| $-3 \leq 2x - 1$	$2x - 1 < 5$
$-2x \geq -1$	$2x < 6$
	$x < 3$

Solve $\quad x^2 - 2x - 8 = 0 \quad$ and

hence $\quad x^2 - 2x - 8 < 0$

$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x = 4 \quad x = -2$$

$y = x^2 - 2x - 8$

$y > 0$

Sleep $\rightarrow$ True

$y < 0$

$$-2 < x < 4$$
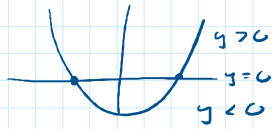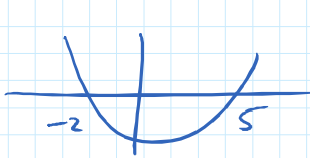
Solve $\quad x^2 - 3x - 10 > 0$

$$x^2 - 3x - 10 = 0 \quad \text{①}$$
$$(x - 5)(x + 2) = 0 \quad \text{②}$$
$$x = 5 \qquad x = -2 \quad \text{③}$$

$y > 0$
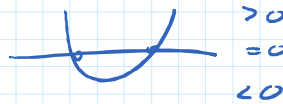$y = 0$
$y < 0$

Solve $\quad x^2 - 3x > 0$

> 0
$= 0$
< 0

$$x^2 - 3x = 0$$
$$x(x - 3) = 0$$
$$x = 0 \qquad x = 3$$

$$0 > x > 3$$

Smaller $\qquad x \qquad$ Bigger

Last Sign.

$$x^2 > 0$$

Solve $\quad x^2 - 9 < 0$

$x^2 < 9$
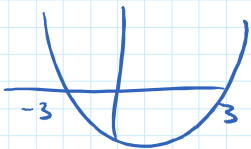$x < 3$

$x^2 = 9$
$x = 3$
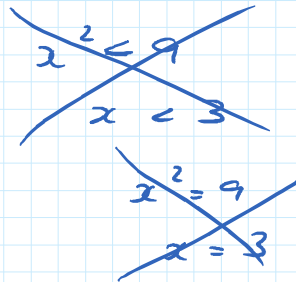
$$x^2 - 9 = 0$$
$$(x - 3)(x + 3) = 0$$
$$x = 3 \qquad x = -3$$
$$-3 < x < 3$$

Solve $\quad 5x - x^2 < 0$

$$x^2 - 5x > 0$$
$$x^2 - 5x = 0$$
$$x = 0 \qquad x = 5$$
$$0 > x > 5$$

Solve (1) $\dfrac{2x - 1}{3} < 5$

$$2x - 1 < 15$$
$$2x < 16$$

$$x \geq 8$$

(11) $\qquad \dfrac{2x-1}{-3} \; \angle \; 5$

$$2x - 1 \; > \; -15$$

$$2x \; > \; -14$$

$$x \; > \; -7$$

Solve $\qquad \dfrac{x+3}{x+2} \; \leq \; 2 \qquad x \in R \quad x \neq -2$

$$\dfrac{x+3}{x+2} \qquad \leq \quad 2$$

$$x + 3 \; \leq \; 2(x+2) \qquad ??$$

$$\dfrac{(x+3)}{x+2}\,(x+2)^2 \; \leq \; 2(x+2)^2$$

$$x^2 + 2x + 3x + 6 \; \leq \; 2(x^2 + 4x + 4)$$

$$x^2 + 5x + 6 \; \leq \; 2x^2 + 8x + 8$$

$$-x^2 - 3x - 2 \; \leq 0$$

$$x^2 + 3x + 2 \; \geq \; 0$$

$$x^2 + 3x + 2 = 0$$

$$(x+1)(x+2) \; = 0$$

$$x = -1 \qquad x = -2$$

$$-2 \geq x \geq -1$$

$$-2 > x \geq -1$$

cannot $\quad = -2$

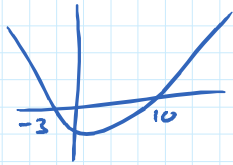Solve $\qquad \dfrac{2x-7}{x+3} \; \leq \; 1 \qquad x \in R \quad x \neq -3$

$$\dfrac{2x-7}{x+3}\,(x+3)^2 \; \leq \; (x+3)^2$$

$$2x^2 + 6x - 7x - 21 \; \leq \; x^2 + 6x + 9$$

$$2x^2 - x - 21 \; \leq \; x^2 + 6x + 9$$

$$x^2 - 7x - 30 \; \leq \; 0$$

$$x^2 - 7x - 30 = 0$$
$$(x - 10)(x + 3) = 0$$
$$x = 10 \qquad x = -3$$
$$-3 \;\textcircled{$\leq$}\; x \leq 10$$
$$-3 < x \leq 10 \;,\quad x \neq -3$$

Solve $\qquad \dfrac{2x-3}{x-5} \;\leq\; \dfrac{3}{2} \qquad x \neq 5$
$$\qquad\qquad\qquad\qquad\qquad\qquad x \in R.$$

$$\frac{2(2x-3)(x-5)^2}{x-5} \leq 3(x-5)^2$$
$$(4x-6)(x-5) \leq 3(x^2 - 10x + 25)$$
$$4x^2 - 20x - 6x + 30 \leq 3x^2 - 30x + 75$$
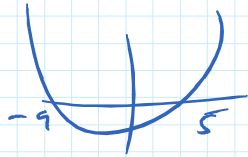$$4x^2 - 26x + 30 \leq 3x^2 - 30x + 75$$
$$x^2 + 4x - 45 \leq 0$$
$$x^2 + 4x - 45 = 0$$
$$(x + 9)(x - 5) = 0$$
$$x = -9 \qquad x = 5$$
$$-9 \leq x < 5$$

Solve $\qquad \dfrac{x}{2x-1} < 1 \qquad x \in R \quad x \neq \frac{1}{2}$
$$\left[\frac{x}{2x-1}(2x-1)^2 \leq (2x-1)^2 \right] \; \text{Wrong} \qquad \text{and} \quad 2x-1 < 0$$

Bottom
$$2x - 1 < 0$$

Bottom is negative $\qquad$ change sign.
$$x > 2x - 1$$
$$-x > -1$$
$$x < 1$$

Solve $\qquad \dfrac{3x}{x-1} < 2 \qquad$ where $\quad x \in R$
$$\qquad\qquad\qquad\qquad\qquad\qquad x \neq 1 \quad \text{and}$$
$$3x < 2(x-1) \qquad\qquad x - 1 > 0$$
$$3x < 2x - 2$$
$$x < -2$$

Solve $\quad \dfrac{2x+1}{x+2} \quad < \dfrac{1}{2} \quad$ where $\quad x \in \mathbb{R}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x \neq -2.$

$$\dfrac{2(2x+1)(x+2)^{\cancel{2}}}{\cancel{x+2}} < \dfrac{1}{2}(x+2)^2$$

$$2(2x^2 + 4x + x + 2) < x^2 + 4x + 4$$

$$4x^2 + 10x + 4 < x^2 + 4x + 4$$
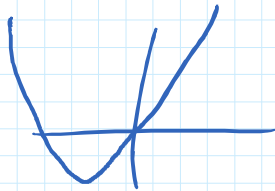
$$3x^2 + 6x < 0$$

$$x^2 + 2x < 0$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$x = 0 \qquad x = -2$$

$$-2 < x < 0$$

Modulus and Inequalities.

Solve $\qquad |x - 1| < 3$

Method 1 $\quad$ Use $\quad +/-$

$$x - 1 < \pm 3 \qquad \text{Wrong}$$

$$\pm(x - 1) < 3$$

$$x - 1 < 3 \qquad\qquad -(x-1) < 3$$

$$x < 4 \qquad\qquad\qquad x - 1 > -3$$

$$\qquad\qquad\qquad\qquad x > -2$$

Method 2: $\quad$ Square

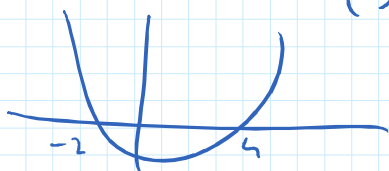$$x^2 - 2x + 1 < 9$$

$$x^2 - 2x - 8 < 0$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x = +4 \qquad x = 2$$

$$-2 < x < 4$$

Method 3: Diagram.
$$|x-1| < 3$$

$$y = 3 \qquad\qquad y = |x-1|$$
$$y = x-1$$
$$x = 0 \quad y = -1$$
$$y = 0 \quad x = 1$$

$$-2 < x < 4$$
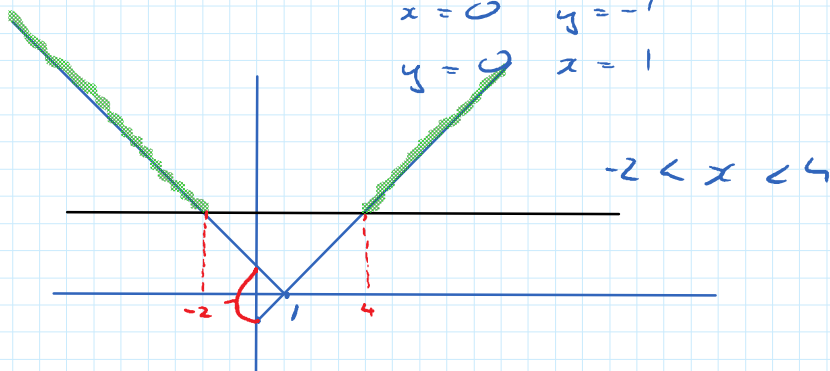
Solve $\qquad |x+2| < 4 \qquad$ Square

$$x^2 + 4x + 4 < 16$$
$$x^2 + 4x - 12 < 0$$
$$x^2 + 4x - 12 = 0$$
$$(x-2)(x+6) = 0$$
$$x = 2 \qquad x = -6$$
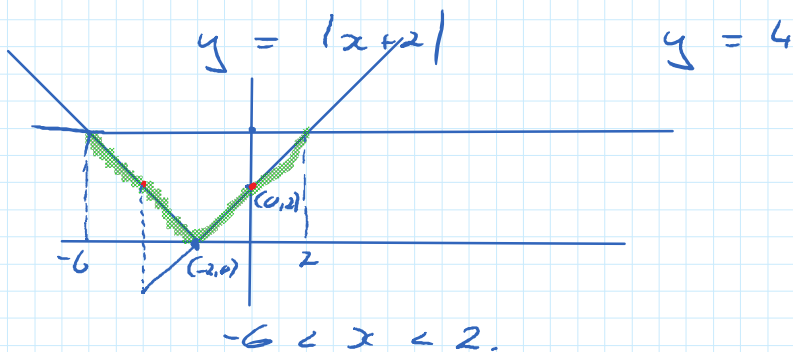$$-6 < x < 2$$

Method 2:
$$|x+2| < 4$$
$$x + 2 < 4 \qquad\qquad or \quad -(x+2) < 4$$
$$x < 2 \qquad\qquad\qquad\qquad x+2 > -4$$
$$x > -6$$

Method $\qquad |x+2| < 4$

$$y = |x+2| \qquad\qquad y = 4$$

$$y = |x+2|$$
$$y = x+2$$
$$x = 0 \qquad y = 2$$
$$y = 0 \qquad x+2 = 0$$
$$x = -2$$

$(0,2)$

-6 $\quad (-2,0) \quad$ 2

$$-6 < x < 2.$$

Abstract Inequalities.

Prove $\qquad a^2 + b^2 \geqslant 2ab$

$$a^2 - 2ab + b^2 \geqslant 0$$

$$(a-b)^2 \geqslant 0$$

Any number squared is positive

Prove $\qquad x^2 + 25 \geqslant 10x$

$$x^2 - 10x + 25 \geqslant 0$$

$$(x-5)^2 \geqslant 0$$

Any
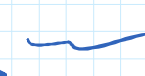
Solve $\qquad x^2 - 6x + 10 \geqslant 0$

$$x^2 - 6x + 9 + 10 - 9 \geqslant 0$$

$$(x-3)^2 + 1 \geqslant 0$$

Sum $\qquad$ 2 numbers. Any

.

(1) Solve

$$\frac{2x-3}{x-1} \leqslant 1 \qquad \text{where} \qquad x \in R, x \neq 1$$

(11) Solve

$$\frac{2x-3}{x-1} \leqslant 1 \qquad \text{where} \qquad x \in R, x \neq 1, x-1 < 0$$

(1) $\qquad \dfrac{2x-3}{x-1} \leqslant 1$

$$\frac{2x-3}{x-1}(x-1)^2 \leqslant (x-1)^2$$

$$2x^2 - 2x - 3x + 3 \leqslant x^2 - 2x + 1$$

$$2x^2 - 5x + 3 \leqslant x^2 - 2x + 1$$

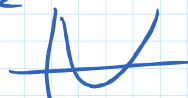$$x^2 - 3x + 2 \leqslant 0$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x - 1 = 0 \qquad x - 2 = 0$$

$$x = 1 \qquad x = 2$$

$$1 < x \leqslant 2$$

(II) $\dfrac{2x-3}{x-1} \leq 1$ $x - 1 < 0$

$2x - 3 \geq x - 1$

$x \geq 2$

Prove $a^2 + 12a + 45 \geq 0$

$a^2 + 12a + 36 + 45 - 36 \geq 0$

$(a + 6)^2 + 9 \geq 0$

Sum of 2 terms one is a square. Any number squared is positive.

Prove $x^2 - 2xy + 2y^2 \geq 0$

$x^2 - 2xy + y^2 + 2y^2 - y^2 \geq 0$

$(x - y)^2 + y^2 \geq 0$

Sum of 2 squares. Any

Prove

$a^3 + b^3 \geq a^2 b + ab^2$ where $a + b > 0$

$a^3 + b^3 - a^2 b - ab^2 \geq 0$

$(a + b)(a^2 - ab + b^2) - ab(a + b) > 0$

$(a + b)(a^2 - ab + b^2 - ab) \geq 0$

$a + b > 0$ in question.

$a^2 - 2ab + b^2 \geq$

$(a - b)^2 \geq 0$

$a^3 + b^3 \geq a^2 b + ab^2$

$(a + b)(a^2 - ab + b^2) \geq ab(a + b)$