Exponential.

Find $\frac{dy}{dx}$ when

(i) $y = (7x-1)e^{x}$ $x = 10$

$\frac{dy}{dx} = (7x-1)e^{x} + 7e^{x}$

(ii) $y = x^{8}e^{x}$

$\frac{dy}{dx} = x^{8}e^{x} + 8x^{7}e^{x}$

$e^{2} \to e^{x}$

(iii) $y = e^{x^2+3}$ $x = 10$

$\frac{dy}{dx} = 2x\,e^{x^2+3}$ Chain

(iv) $y = e^{5x+9}$

$\frac{dy}{dx} = 5e^{5x+9}$

(v) $y = e^{13x+14}$

$\frac{dy}{dx} = 13e^{13x+14}$

(vi) $y = e^{x^3+5x}$

$\frac{dy}{dx} = (3x^2+5)e^{x^3+5x}$

Differ the power by differ the e
but e stays the same.

(vi) $y = \frac{e^{x}}{x}$

$u = e^{x}$ $v = x$

$\frac{du}{dx} = e^{x}$ $\frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{xe^x - e^x}{x^2}$$

(vii) $$y = \frac{e^{x^2}}{e^{7x}}$$ Indices

$$y = e^{x^2 - 7x}$$

$$\frac{dy}{dx} = (2x-7)e^{x^2-7x}$$

(viii) $$y = e^{x^3} \cdot e^{6x}$$

$$y = e^{x^3+6x}$$

$$\frac{dy}{dx} = (3x^2+6)e^{x^3+6x}$$

$x = 10$

Logs

$$y = x \ln x$$

$$\frac{dy}{dx} = x\frac{1}{x} + \ln x$$

$$= 1 + \ln x$$

$$y = \frac{\ln x}{x^3}$$

$u = \ln x$ $\qquad$ $v = x^3$

$\frac{du}{dx} = \frac{1}{x}$ $\qquad$ $\frac{dv}{dx} = 3x^2$

$$\frac{dy}{dx} = \frac{x^3\frac{1}{x} - 3x^2\ln x}{(x^3)^2}$$

$$= \frac{x^2 - 3x^2 \ln x}{x^6}$$

by = prod

df = chan

$x = 10$

$\ln x \to \frac{1}{x}$

$$y = \ln(7x+3)$$

$$\frac{dy}{dx} = 7 \cdot \frac{1}{7x+3}$$

$$= \frac{7}{7x+3}$$

$$y = \ln(x^2 + 8x)$$

$$\frac{dy}{dx} = 2x+8 \cdot \frac{1}{x^2+8x}$$

$$= \frac{2x+8}{x^2+8x}$$

$$y = \ln(9x+3)$$

$x = 10$

$$\frac{dy}{dx} = 9 \cdot \frac{1}{9x+3} = \frac{9}{9x+3}$$

Diff the bracket by diff the log but log becomes one over.

Rules of Logs

$$y = \ln\left(\frac{3x+5}{2x-9}\right)$$

$$y = \ln(3x+5) - \ln(2x-9)$$

$$\frac{dy}{dx} = \frac{3}{3x+5} - \frac{2}{2x-9}$$

$$y = \ln(3x^2+7)^5$$

$$\cancel{y = 6x(5)\frac{1}{(3x^2+7)^4}}$$

$$\frac{dy}{dx} = 6x(5)(3x^2+7)^4\frac{1}{(3x^2+7)^5}$$

$\log a^n = n\log a$

$$y = 5\ln(3x^2+7)$$

$$\frac{dy}{dx} = 5 \cdot \frac{6x}{3x^2+7} = \frac{30x}{3x^2+7}$$

$$y = \ln\sqrt{\frac{5x+3}{7x+8}}$$

$\sqrt{x} = x^{\frac{1}{2}}$

$$y = \ln\left(\frac{5x+3}{7x+8}\right)^{\frac{1}{2}}$$

$$y = \frac{1}{2}\ln\left(\frac{5x+3}{7x+8}\right)$$

$\ln x \to \frac{1}{x}$

$$y = \frac{1}{2}\left[\ln(5x+3) - \ln(7x+8)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left\{5 \cdot \frac{1}{5x+3} - 7 \cdot \frac{1}{7x+8}\right]$$

$$y = \ln\sqrt{\frac{8x+3}{5x-1}}$$

$$y = \ln\left(\frac{8x+3}{5x-1}\right)^{\frac{1}{2}}$$

$$y = \frac{1}{2}\ln\left(\frac{8x+3}{5x-1}\right)$$

$$y = \frac{1}{2}\left[\ln(8x+3) - \ln(5x-1)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[8 \cdot \frac{1}{8x+3} - 5 \cdot \frac{1}{5x-1}\right]$$

Find $\frac{dy}{dx}$ when

$e^x \to e^x$

(i) $y = e^{9x+4}$ Algebra $x=1$

$$\frac{dy}{dx} = 9e^{9x+4}$$

$9(1)+4 = 13$

e^{13}

(ii) $y = \ln(3x+1)$

$$\frac{dy}{dx} = 3 \cdot \frac{1}{3x+1} = \frac{3}{3x+1}$$

(iii) $y = (7x^2+5)^{10}$ $x=1$

$$\frac{dy}{dx} = 14x(10)(7x^2+5)^9$$

(iv) $y = x(2x+1)^7$ by = prod

$$\frac{dy}{dx} = x(2)(7)(2x+1)^6 + (2x+1)^7$$

$$= 14x(2x+1)^6 + (2x+1)^7$$

$$= (2x+1)^6(14x + 2x+1)$$

$$= (16x+1)(2x+1)^6$$

(v) $y = x^2 \sin(6x+5)$

$$\left[\frac{dy}{dx} = 2x\, 6(\cos(6x+5)\right]$$ by = prod $\sin(6x+5)$

$$\frac{dy}{dx} = x^2(6)\cos(6x+5) + 2x\sin(6x+5)$$

$$= 6x^2\cos(6x+5) + 2x\sin(6x+5)$$

$$y = x^8 \sin x$$

$$\frac{dy}{dx} = x^8\cos x + 8x^7\sin x$$

$$y = \sin(x^3+5x)$$

$$\frac{dy}{dx} = (3x^2+5)\cos(x^3+5x)$$

$$y = \sin^6 x$$ Learn

$$y = (\sin x)^6$$

$$\frac{dy}{dx} = \cos x\, 6(\sin x)^5$$

$$= 6\cos x \sin^5 x$$

$y = \sin^3(5x+1)$

$y = [\sin(5x+1)]^3$

$x = 1$
$5x + 1$
$\sin(5x+1)$
$[\sin(5x+1)]^3$

$\frac{dy}{dx} = 5\cos(5x+1)\,3(\sin(5x+1))^2$

$= 15\cos(5x+1)\sin^2(5x+1)$

Inverse Trig.

Find $\frac{dy}{dx}$ when

$y = \tan^{-1}\frac{x}{5}$

$\tan^{-1}\frac{x}{a} \rightarrow \frac{a}{a^2+x^2}$

$y = \tan^{-1}\frac{x}{5}$

$\frac{dy}{dx} = \frac{5}{5^2+x^2} = \frac{5}{25+x^2}$

$a =$ constant

$y = \sin^{-1}\frac{x}{7}$

$\sin^{-1}\frac{x}{a} \rightarrow \frac{1}{\sqrt{a^2-x^2}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{49-x^2}}$

$$y = \tan^{-1} x$$

$$y = \tan^{-1} \frac{x}{1}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$y = \tan^{-1} 10x$$

$$y = \tan^{-1} \frac{10x}{1}$$

$x = 1$

$10x$

$\tan^{-1}$

$$\frac{dy}{dx} = 10 \cdot \frac{1}{1+(10x)^2}$$

$$\frac{10}{1+100x^2}$$

$$y = x \tan^{-1} \frac{x}{1}$$

$$\frac{dy}{dx} = x \cdot \frac{1}{1+x^2} + \tan^{-1} x$$

$$= \frac{x}{1+x^2} + \tan^{-1} x$$

$$y = \sin^{-1}(3x-2)$$

$x = 5$

$$\frac{dy}{dx} = 3 \cdot \frac{1}{\sqrt{1^2 - (3x-2)^2}}$$

$3x-2$

$\sin^{-1}\left(\frac{3x-2}{1}\right)$

$a = 1 \quad x = 3x-2$

$$y = \tan^{-1}(5x+1)$$

$$y = \tan^{-1}\left(\frac{5x+1}{1}\right) \qquad x = 10$$

$$\frac{dy}{dx} = 5 \cdot \frac{1}{1+(5x+1)^2}$$