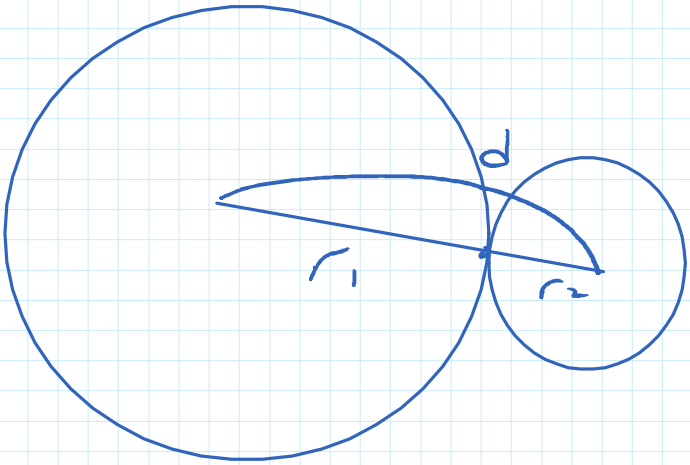
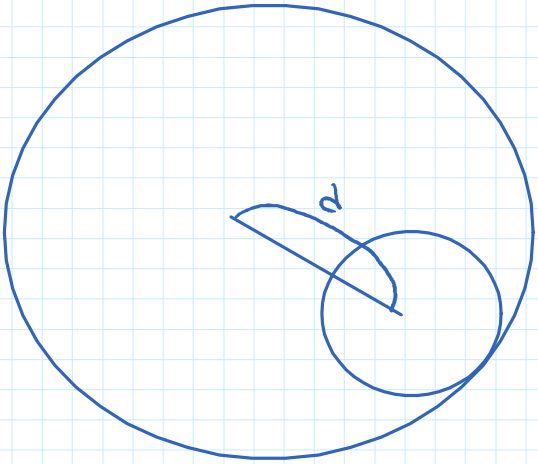
Touching Circles.



Externally

$r_1 + r_2 = d$.

d = distance between centres



Internally

$d = r_2 - r_1$

Prove $x^2+y^2-2x-15=0$ and $x^2+y^2-14x-16y+77=0$ touch externally and find point of contact.

$x^2+y^2-2x-15=0$

Centre $(1,0)$ $\quad r = \sqrt{1+15} = 4$

$x^2+y^2-14x-16y+77=0$

Centre $(7,8)$ $\quad r = \sqrt{49+64-77}$

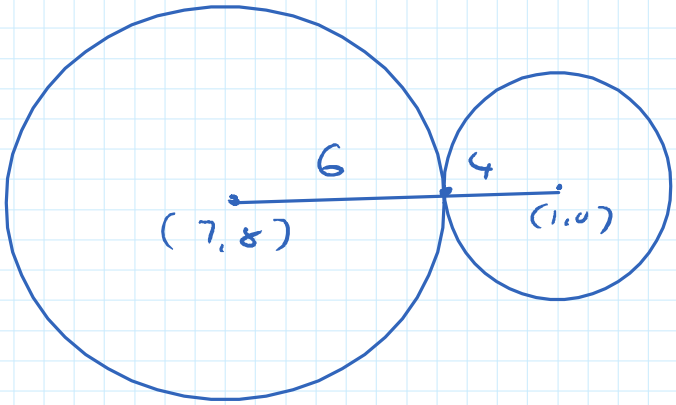
$= 6$

Distance (1,0) (7,8)

$$\sqrt{(7-1)^2+(8-0)^2} = \sqrt{36+64}$$

$$= \sqrt{100} = 10$$

$$10 = 6 + 4$$



(7,8) x_1, y_1 (1,0) $x_2 y_2$ 3:2 a b

$$\left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + by_2}{b+a}\right)$$

$$\left(\frac{14+3}{5}, \frac{16+0}{5}\right) = \left(\frac{17}{5}, \frac{16}{5}\right)$$

Prove $x^2+y^2+4x-6y+12=0$ and $x^2+y^2-12x+6y-76=0$ touch internally.

$$x^2+y^2+4x-6y+12=0$$

Centre $(-2, 3)$

$$r = \sqrt{4+9-12}$$

$$= 1$$

$$x^2+y^2-12x+6y-76=0$$

Centre $(6, -3)$

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$$r = \sqrt{36 + 9 + 76} = 11$$

Distance $(-2, 3)$ $(6, -3)$

$$= \sqrt{(6+2)^2 + (-3-3)^2}$$

$$\sqrt{64 + 36} = \sqrt{100}$$

$$= 10 = 11 - 1$$