Cubics

Let $P(z) = z^3 - kz^2 + 22z - 20,\ k \in \mathbf{R}$.

$3 + i$ is a root of the equation $P(z) = 0$.
Find the value of k.
Find the other two roots of the equation $P(z) = 0$.

Cubic $\Rightarrow$ Factor Theorem
$\Rightarrow$ Sub in.

Note: $k \in R \Rightarrow k$ is real
$\Rightarrow$ roots will occur in conjugate pair.

Sub in $3+i$

$$z^3 - kz^2 + 22z - 20 = 0$$

$$(3+i)^3 - k(3+i)^2 + 22(3+i) - 20 = 0$$

$$(3+i)^2 = 9 + 6i + i^2 = 8 + 6i$$

$$(3+i)^3 = (3+i)(3+i)^2$$
$$= (3+i)(8+6i)$$
$$= 24 + 18i + 8i + 6i^2$$
$$= 18 + 26i$$

$$18 + 26i - k(8+6i) + 66 + 22i - 20 = 0 + 0i$$

$$64 + 48i - 8k - 6ki = 0 + 0i$$

Real = Real $\quad 64 - 8k = 0$

$$k = 8$$

One root $3+i$

Other is $3-i \Rightarrow$ conjugate root theorem.

In a cubic if all coefficient are real then 2 roots will occur in conjugate pairs

A second root $3-i$.

Can now form a quadratic

Sum $3+i + 3-i = 6$

Prod $(3+i)(3-i) = 9-i^2 = 10$

$$z^2 - 6z + 10$$

$$\begin{array}{r} z - 2 \\ z^2 - 6z + 10 \overline{\smash{)}\, z^3 - 8z^2 + 22z - 20} \end{array}$$

$z - 2 = 0$

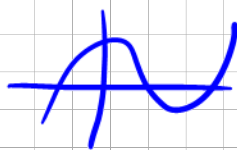
$z = 2$ is 3^{rd} root.

First into first

Last into last

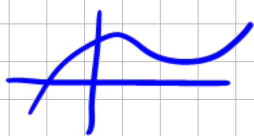
For a cubic we have

(i) 3 real distinct roots



(ii) 3 real roots 2 the same

(iii) 1 real and 2 imaginary



Imaginary are conjugates if coefficients of cubic are all real (conjugate root theorem).