

Binomial or Bernoulli Trials

Toss a coin 3 times. Find prob of getting 2 tails?

$$T \quad T \quad H \\ \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} \times \frac{3!}{2!} = 3 = \binom{3}{2}$$

Coin 5 times prob of 2 tails?

$$T \quad T \quad H \quad H \quad H \\ \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 \frac{5!}{3!2!} = 10 = \binom{5}{2}$$

↑ ↑
H T

Conditions.

- (i) Fixed number, n , of repeated trials
- (ii) Only two possible outcomes
⇒ success (p) or failure (q)
- (iii) Trials are independent.
- (iv) The prob of success in each trial is a constant.

r = number of required results.

$$P(r) = \binom{n}{r} p^r q^{n-r}$$

Coin 3 times 2 tails.

$$p(t) = p = \frac{1}{2} \quad p(H) = q = \frac{1}{2}$$

$n = 3 \quad r = 2$

$$\binom{3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1$$

Note. Use Bernoulli when we do not know the order.

The prob I score a free is $\frac{3}{5}$. In 6 frees what is prob I

(1) score twice?

$$p = \frac{3}{5} \quad q = \frac{2}{5} \quad n = 6 \quad r = 2$$

$$\begin{aligned} P(r) &= \binom{n}{r} p^r q^{n-r} \\ &= \binom{6}{2} \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^4 \\ &= \frac{432}{3125} \end{aligned}$$

(ii) Scores 4 times? EKKER

$$p = \frac{3}{5} \quad q = \frac{2}{5} \quad n = 6 \quad r = 4$$

$$P(4) = \binom{6}{4} \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^2 = \frac{972}{3125}$$

(iii) Scores only first and last free?

Not Bernoulli = know order

$$\left[p = \frac{3}{5} \quad q = \frac{2}{5} \quad n = 6 \quad r = 1 \right]$$

$$\frac{3}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} = \frac{144}{15625}$$

(iv) Scores for 3rd time on 6th free?

Has to score 6th \Rightarrow must score 2 of first 5

$$p = \frac{3}{5} \quad q = \frac{2}{5} \quad n = 5 \quad r = 2$$

$$\binom{5}{2} \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^3 \cdot \frac{3}{5} = \frac{432}{3125}$$

Score 6th

A test consists of twenty multiple-choice questions. Each question has four possible answers, only one of which is correct. Seán decides to guess all the answers at random.

Find the probability that:

- (i) Seán gets none of the answers correct
- (ii) Seán gets exactly five of the answers correct
- (iii) Seán gets four, five or six of the answers correct.

Give each of your answers correct to three decimal places.

$$\begin{aligned} p &= \frac{1}{4} & q &= \frac{3}{4} & n &= 20 & r &= 0 \\ (i) \quad & {}^n C_r p^r q^{n-r} &= & {}^{20} C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{20} \\ & & & & & & &= 0.003 \end{aligned}$$

$$(ii) \quad r = 5 \quad {}^{20} C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{15} = 0.202$$

$$\begin{aligned} (iii) \quad & r = 4 & r = 5 & r = 6 \\ & {}^{20} C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{16} & & + \\ & {}^{20} C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{15} & & + \\ & & & \frac{0.202}{\hline} \\ & & & 0.561 \end{aligned}$$