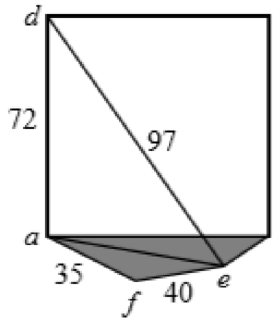
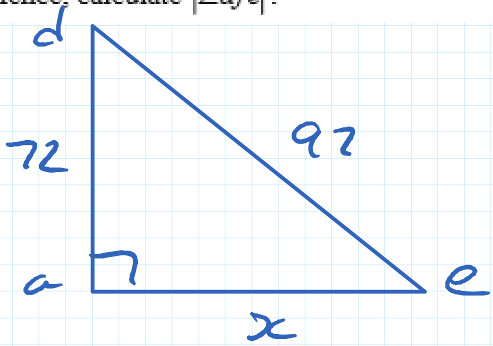
a, f and e are points on horizontal ground.
d is a point on a vertical wall directly above a.

$|ad| = 72$ m, $|de| = 97$ m,
$|af| = 35$ m and $|fe| = 40$ m.

(i) Calculate $|ae|$.

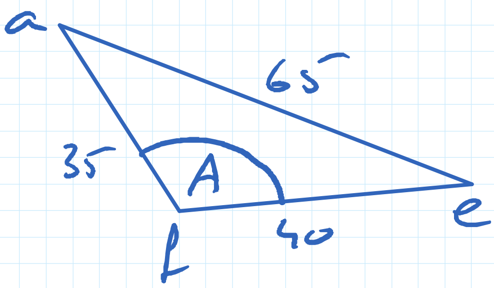
(ii) Hence, calculate $|\angle afe|$.





$x^2 + 72^2 = 97^2$

$x = 65$



$a^2 = b^2 + c^2 - 2bc \cos A$

$65^2 = 35^2 + 40^2 - 2(35)(40)\cos A$

$2800 \cos A = -1400$

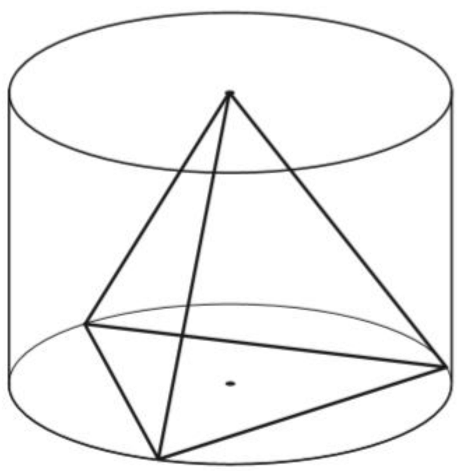
$\cos A = -\frac{1}{2}$

$A = 120°$

A regular tetrahedron has four faces, each of which is an equilateral triangle.

A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron. The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom.

If the length of one edge of the tetrahedron is $2a$, show that the volume of the smallest possible cylindrical container is $\left(\frac{8\sqrt{6}}{9}\right)\pi a^3$.



$\frac{1}{2} + \frac{2}{3}$

Read question twice before we start.

A regular tetrahedron has four faces, each of which is an equilateral triangle.

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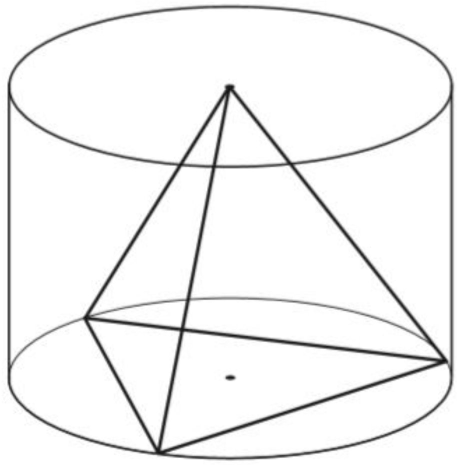
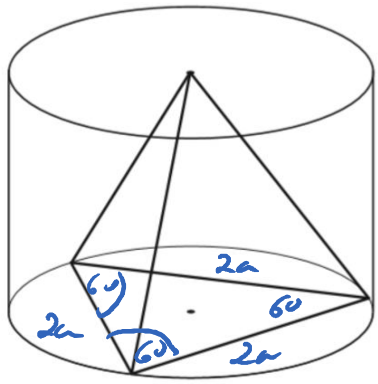
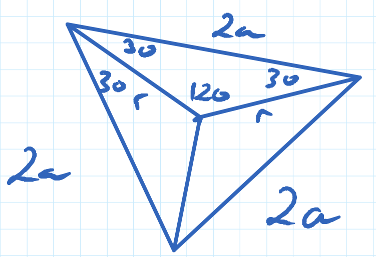
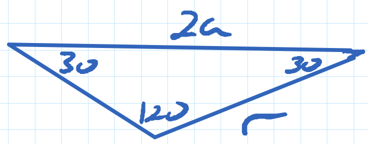




$$V = \pi r^2 h.$$

Need r and h in terms of a.

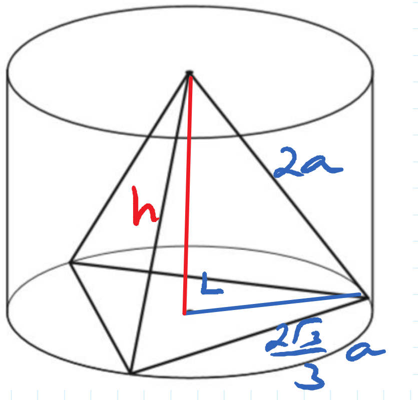


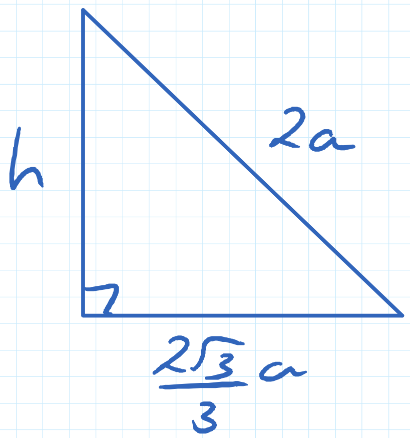


$$\frac{r}{\sin 30} = \frac{2a}{\sin 120}$$

$$r = \frac{2a \sin 30}{\sin 120}$$

$$= \frac{2\sqrt{3}}{3} a$$





$$h^2 + \left(\frac{2\sqrt{3}}{3}\right)^2 = (2a)^2$$

$$h^2 = \frac{8}{3}a^2$$

$$h = \frac{2\sqrt{2}}{\sqrt{3}}a$$

$$V = \pi r^2 h$$

$$= \pi\left(\frac{2\sqrt{3}a}{3}\right)^2 \frac{2\sqrt{2}}{\sqrt{3}}a$$

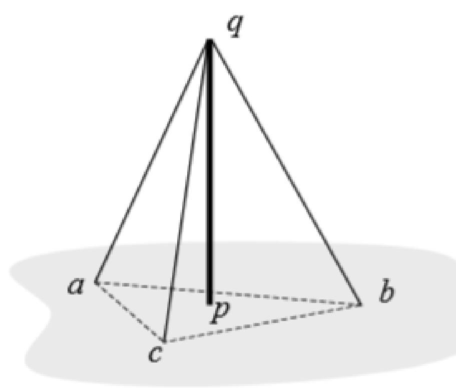
$$= \frac{8\sqrt{6}}{9}a^3\pi$$

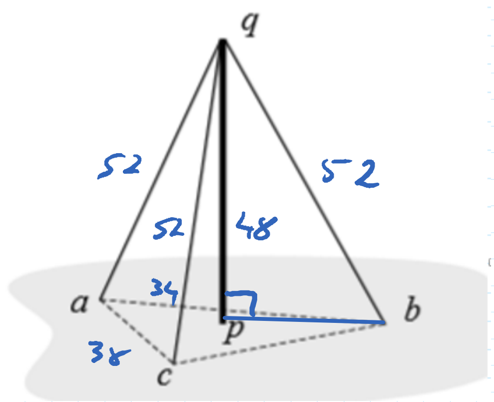
A vertical radio mast [pq] stands on flat horizontal ground. It is supported by three cables that join the top of the mast, q, to the points a, b and c on the ground. The foot of the mast, p, lies inside the triangle abc.

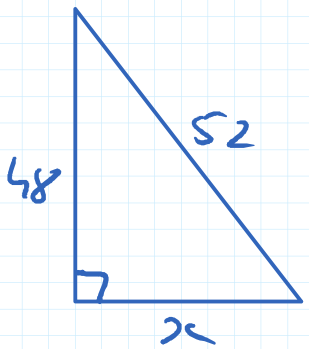
Each cable is 52 m long and the mast is 48 m high.

(i) Find the (common) distance from p to each of the points a, b and c.

(ii) Given that $|ac| = 38$ m and $|ab| = 34$ m, find $|bc|$ correct to one decimal place.

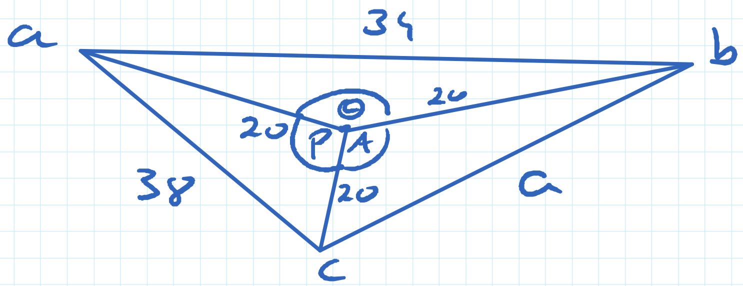






$$x^2 + 48^2 = 52^2$$

$$x = 20$$



$$38^2 = 20^2 + 20^2 - 2(20)(20)\cos P$$

$$800\cos P = 400 + 400 - 1444$$

$$\cos P = -\frac{161}{200}$$

$$P = 143.6^\circ$$

$$34^2 = 20^2 + 20^2 - 2(20)(20)\cos Q$$

$$800\cos Q = 400 + 400 - 1156$$

$$\cos Q = \frac{-89}{200}$$

$$\theta = 116.4^\circ$$

$$A = 360 - (143.6 + 116.4)$$

$$A = 100^\circ$$

$$a^2 = 20^2 + 20^2 - 2(20)(20)\cos 100$$

$a = 30.5\ \text{cm}.$