I am 25 years old. I will retire at 65 years old. I will die at 87 years old. I will pay €450 per month at start of month into my pension. How much will my pension be at end of each month given discount rate is 2.3% on average per annum?

Pay €450 per month

Age 25 → 65 = 40 × 12 = 480

Pension = F = lump

$(1+c)^{12} = 1.023$

$= 1.001$

$F = P(1+c)^t$

$450(1.001)^{480} + 450(1.001)^{479} + \cdots 450(1.001)$

$450\left[1.001 + 1.001^2 + \cdots\right]$

$a = 1.001 \quad r = 1.001 \quad n = 480$

$S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{1.001(1-1.001^{480})}{1-1.001}$

$= 783$

$F = €352,571.40$. Pen put

65 → 87 ⇒ 22 × 12 = 264.

$P = 352,571.40 \quad F = ??$

$P = \frac{F}{(1+c)^t}$

$$352,571.40 = \frac{F}{1.001} + \frac{F}{(1.001)^2} + \cdots$$

$$= F\left[\frac{1}{1.001} + \frac{1}{1.001^2} + \cdots\right]$$

$$a = \frac{1}{1.001} \qquad r = \frac{1}{1.001} \qquad n = 264$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{\frac{1}{1.001}\left(1 - \left(\frac{1}{1.001}\right)^{264}\right)}{1 - \frac{1}{1.001}}$$

$$= 207.$$

$$352,257.40 = 207F$$

$$F = €1697.39$$

I start pension at 23 yrs old. Retire at 60 yrs old. I will die at 90 yrs old. I want a pension of €2,000 a month paid at start of month. AER is 3%. How much do I need to pay into pension at start of each month?

$23 \Rightarrow 60 = 37 \times 12 = 444$

$60 \Rightarrow 90 = 30 \times 12 = 360$

$(1+r)^{12} = 1.03$

$1+r = \sqrt[12]{1.03} = 1.002$

Pension = €2000 + €2000

Pension pt = $P = \frac{F}{(1+r)^t}$

Start.

$P = 2000 + \frac{2000}{1.002} + \cdots$

$= 2000\left[1 + \frac{1}{1.002} + \cdots\right]$

$a = 1 \qquad r = \frac{1}{1.002} \qquad n = 360$

$S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{1\left(1-\left(\frac{1}{1.002}\right)^{360}\right)}{1-\frac{1}{1.002}}$

$= 239$

Pension = €478,020.24

Age 23 Pension F = €478,020.24

$$F = P(1+i)^t$$

$$P(1.002)^{444} + P(1.002)^{443} + \cdots + P(1.002) = 478,020.24$$

$$P\left[1.002 + 1.002^2 + \cdots\right] = 478,020.24$$

$$a = 1.002 \quad r = 1.002 \quad n = 444$$

$$S_n = \frac{1.002\left(1-1.002^{444}\right)}{1-1.002}$$

$$= 806$$

$$806P = 478,020.24$$

$$P = €592.39$$

I have a pension pot of €350,000. How much will my pension with AER of 1.5% paid at start of month be if

(i) I live for 25 yrs?

(ii) I live for ever?

$$(1+i)^{12} = 1.015$$

$$1+i = \sqrt[12]{1.015} = 1.001$$

(i) $t = 25 \times 12 = 300$

$$P = 350,000 \qquad P = \frac{F}{(1+i)^t}$$

$$350,000 = F + \frac{F}{1.001} + \frac{F}{(1.001)^2}$$

$$F\left(1 + \frac{1}{1.001} + \cdots\right)$$

$a = 1 \qquad r = \frac{1}{1.001} \qquad n = 300$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{1 - \left(\frac{1}{1.001}\right)^{500}}{1 - \frac{1}{1.001}}$$

$350.00 = 250F$

$\frac{350.00}{250} = F$

$€1396.36$

(11) Line forever $\quad n = \infty$

$$S_\infty = \frac{a}{1 - r} = \frac{1}{1 - \frac{1}{1.001}}$$

$352.00 = 806F$

$433.98 = F$