Quadratics.

Type 1.

Solve $x^2 + 2x + 2 = 0$.

$a = 1 \quad b = 2 \quad c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 8}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2} \qquad \sqrt{-4} = \sqrt{4}\sqrt{-1} = 2i$$

$$= \frac{-2 \pm 2i}{2}$$

$$= -1 \pm i$$

Solve the equation $iz^2 + (2-3i)z + (-5+5i) = 0$.

$a = i \quad b = 2 - 3i \quad c = -5 + 5i$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 + 3i \pm \sqrt{(2-3i)^2 - 4i(-5+5i)}}{2i}$$

$$(2-3i)^2 - 4i(-5+5i)$$

$$4 - 12i + 9i^2 + 20i - 20i^2$$

$$4 + 8i - 11i^2$$

$$15 + 8i$$

Let $\sqrt{15 + 8i} = a + bi$

$$15 + 8i = a^2 + 2abi + b^2 i^2$$

$$15 = a^2 - b^2$$

$$8 = 2ab$$

$$b = \frac{4}{a}$$

$$15 = a^2 - \frac{16}{a^2}$$

$15a^2 = (a^2)^2 - 16$ $\quad t = a^2$

$t^2 - 15t - 16 = 0$

$(t-16)(t+1) = 0$

$t = 16$ $\quad t = -1$

$a^2 = 16$ $\quad a^2 = -1$ ~~$a^2 = -1$~~

$a = \pm 4$

$b = \frac{4}{a}$ $\quad b = \pm 1$

$\sqrt{15 + 8i} = \pm(4 + i)$

$\frac{-2 + 3i \pm (4 + i)}{2i}$

$\frac{-2 + 3i + 4 + i}{2i}$ $\quad \frac{-2 + 3i - 4 - i}{2i}$

$\frac{2 + 4i}{2i}$ $\quad -\frac{6 + 2i}{2i}$

$\frac{1 + 2i}{i} \cdot \frac{i}{i}$ $\quad -\frac{3 + i}{i} \cdot \frac{i}{i}$

$\frac{i + 2i^2}{i^2}$ $\quad -\frac{3i + i^2}{-1}$

$\frac{-2 + i}{-1} = 2 - i$ $\quad -\frac{1 - 3i}{-1} = 1 + 3i$

Type 2:

Form a quadratic with roots $2 \pm 5i$

One root $= 2 + 5i$ $\quad$ Other $= 2 - 5i$

$z^2 - (\text{sum of roots})z + \text{product of roots} = 0$

Sum $= 2 + 5i + 2 - 5i = 4$

Prod $= (2 + 5i)(2 - 5i) = 4 - 25i^2 = 29$

$z^2 - 4z + 29 = 0$

Type 3.

$3+i$ is a root of $z^2+az+bi=0$ find a and b.

Choice 1 $\Rightarrow$ $-b$ formula

Choice 2 $\Rightarrow$ Sum / Prod

Choice 3 $\Rightarrow$ Sub in

Sub in $3+i$

$$(3+i)^2+a(3+i)+bi=0+0i$$

$$9+6i+i^2+3a+ai+bi=0+0i$$

$$8+6i+3a+ai+bi=0+0i$$

$$8+3a=0$$

$$a=-\frac{8}{3}$$

$$6+a+b=0$$

$$\frac{18}{3}-\frac{8}{3}+b=0$$

$$b=\frac{-10}{3}$$