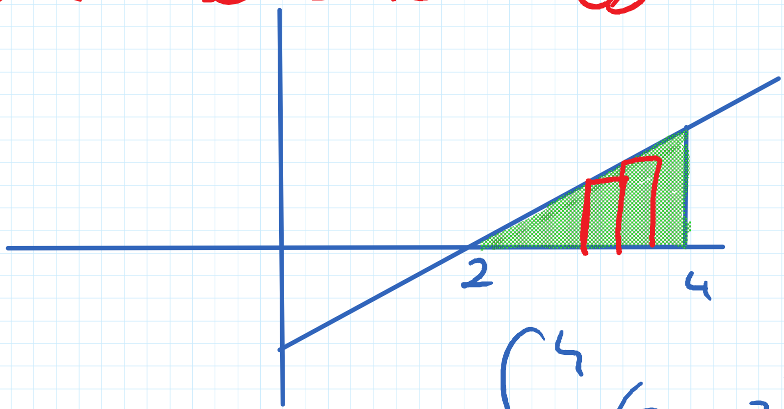
Area under a Curve

Find area under $y = x - 2$ from $x = 2$ to $x = 4$.



$y = x - 2$

$x = 0 \quad y = -2$

$y = 0 \quad x = 2.$

$$\int_2^4 (x-2)\,dx$$

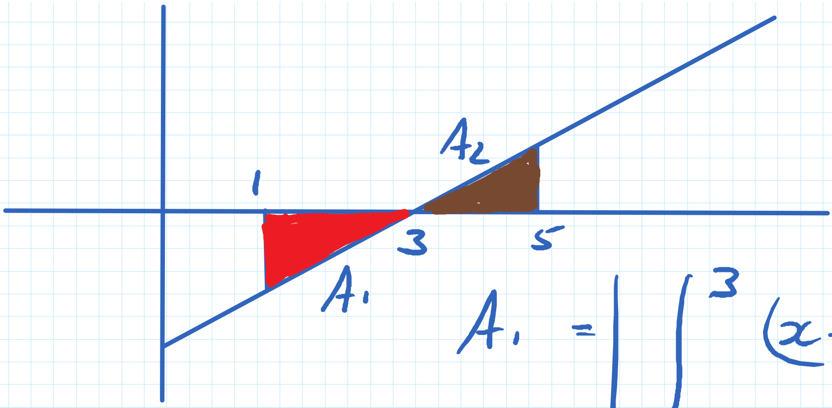
$$\left[\frac{x^2}{2} - 2x\right]_2^4$$

$$\frac{16}{2} - 2(4) - \left(\frac{4}{2} - 4\right)$$

$$8 - 8 - (2 - 4) = 2 \text{ sq units}$$

Area under $y = x - 3$ from $x = 1$ to $x = 5$.

$$\int_1^5 (x-3)\,dx = 0$$



$y = x - 3$

$x = 0 \quad y = -3$

$y = 0 \quad x = 3$

$$A_1 = \left| \int_1^3 (x-3)\, d \right| = \left| \left[\frac{x^2}{2} - 3x \right]_1^3 \right|$$

$= 2$

$$A_2 = \int_3^5 (x-3)\, dx = 2 \quad *$$

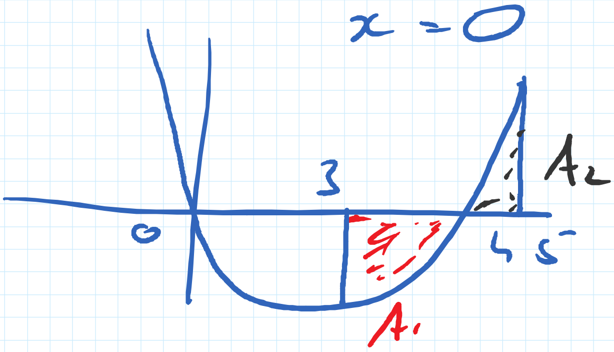
Ans 4 sq units.

Find area between $y = x^2 - 4x$ and x-axis from $x = 3$ to $x = 5$.

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0 \quad x = 4$



$$A_1 = \left| \int_3^4 (x^2 - 4x)\, dx \right|$$

$$\left|\left[\frac{x^3}{3} - \frac{\cancel{4}^{2}x^2}{\cancel{4}}\right]_3^4\right|$$

$$\left|\left[\frac{x^3}{3} - 2x^2\right]_3^4\right|$$

$$\left|\frac{64}{3} - 32 - \left(\frac{27}{3} - 18\right)\right|$$

$$= \frac{5}{3}$$

$$A_2 = \int_4^5 (x^2 - 4x)\,dx$$

$$\left[\frac{x^3}{3} - 2x^2\right]_4^5$$

$$\frac{125}{3} - 50 - \left(\frac{64}{3} - 32\right) = \frac{7}{3}$$

$$\frac{5}{3} + \frac{7}{3} = 4 \text{ sq units.}$$

Double Shapes.

Find the area between $y = x^2$ and $y = x$.

Points of intersection

$$x^2 = x$$

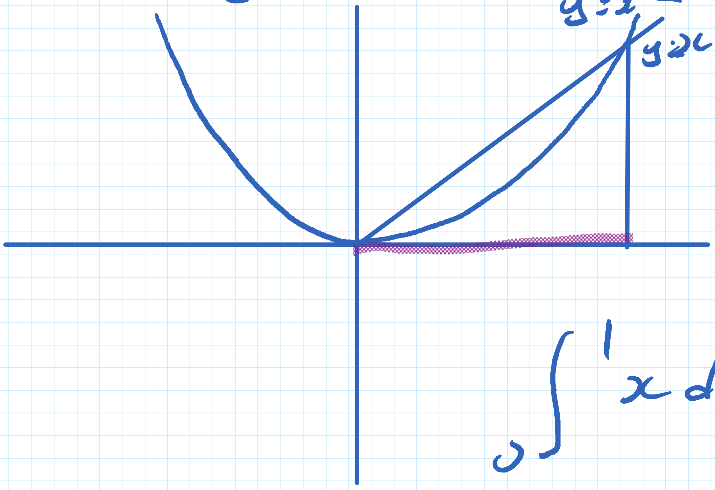
$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0 \qquad x = 1$$

$y = x^2$ $y = x$

$x = 0 \quad y = 0$ $x = 0 \quad y = 0$



$y = x$

$x = 0 \quad y = 0$

$x = 1 \quad y = 1$

$$\int_0^1 x\,dx - \int_0^1 x^2\,dx$$

$$\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6} \text{ sq units}$$

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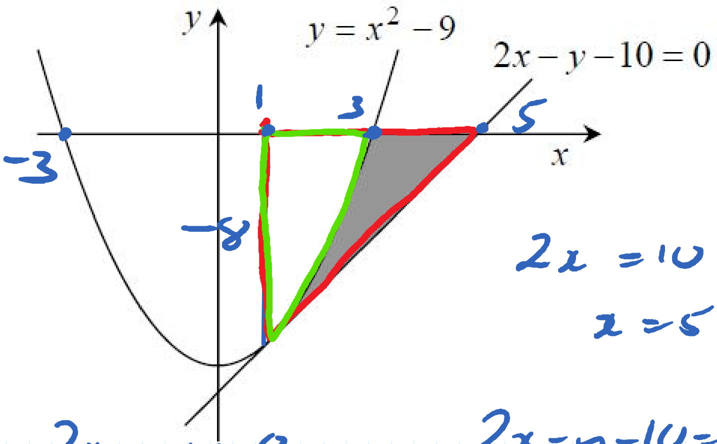
The line $2x - y - 10 = 0$ is a tangent to the curve $y = x^2 - 9$, as shown.

The shaded region is bounded by the line, the curve and the x-axis.

Calculate the area of this region.



$$x^2 - 9 = 0$$
$$x^2 = 9$$
$$x = \pm 3$$

$$2x = 10$$
$$x = 5$$

$$2x - y - 10 = 0$$
$$x = 1 \quad -8 = y$$

$$2x - y - 10 = 0$$
$$2x - 10 = y$$

$$x^2 - 9 = 2x - 10$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)(x-1) = 0$$
$$x - 1 = 0$$
$$x = 1$$

$$\left| \int_1^5 (2x - 10)\,dx - \int_1^3 (x^2 - 9)\,dx \right|$$

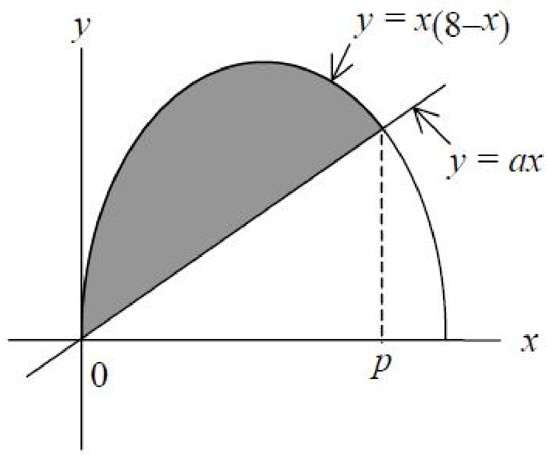
$$\left| -16 - \left[\frac{x^3}{3} - 9x \right]_1^3 \right]$$

a is a real number such that $0 < a < 8$.

The line $y = ax$ intersects the curve $y = x(8 - x)$ at $x = 0$ and at $x = p$.

(i) Show that $p = 8 - a$.

(ii) Show that the area between the curve and the line is $\frac{p^3}{6}$ square units.



$y = x(8-x) \qquad y = ax$

$x(8-x) = ax$

$8 - x = a \quad \Rightarrow a = 8-p$

$x = p \qquad 8 - a = p$

$$\int_0^p (8x - x^2) - \int_0^p ax \, dx$$

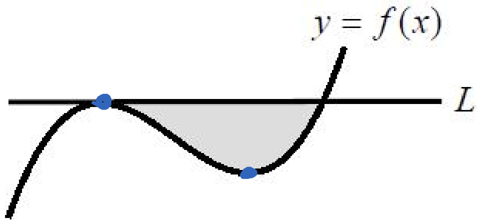
$$\left[\frac{8x^2}{2} - \frac{x^3}{3} - \frac{ax^2}{2} \right]_0^p$$

$$4p^2 - \frac{p^3}{3} - \frac{ap^2}{2}$$

$$4p^2 - \frac{p^3}{3} - \frac{p^2(8-p)}{2}$$

$$\frac{24p^2 - 2p^3 - 3p^2(8-p)}{6} = \frac{p^3}{6}$$

Let $f(x) = x^3 - 3x^2 + 5$.
L is the tangent to the curve $y = f(x)$ at its local maximum point.



Find the area enclosed between L and the curve.

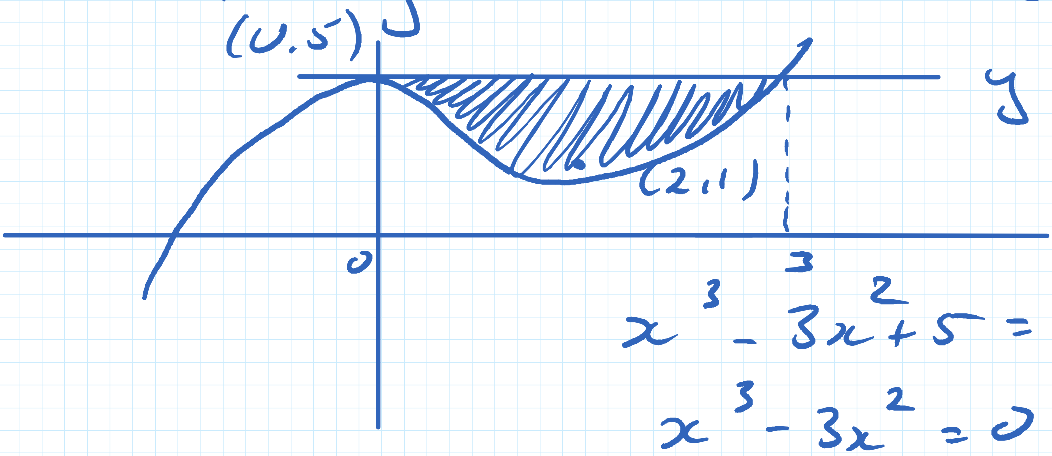
$$y = x^3 - 3x^2 + 5$$

$$\frac{dy}{dx} = 3x^2 - 6x = 0$$

$$x(x-2) = 0 \qquad x = 0 \quad x = 2$$

$$x = 0 \quad y = 5 \quad (0, 5)$$

$$x = 2 \quad y = 8 - 12 + 5 = 1 \quad (2, 1)$$



$y = 5$

$$x^3 - 3x^2 + 5 = 5$$

$$x^3 - 3x^2 = 0$$

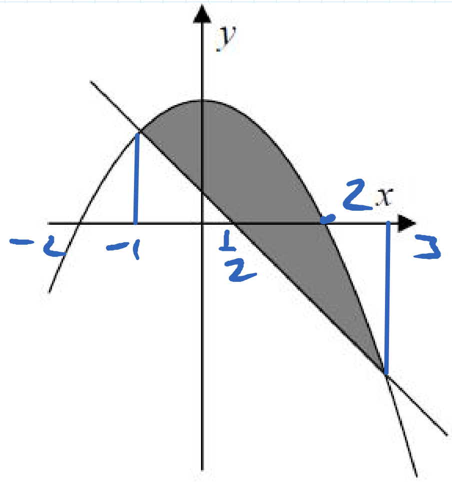
$$x^2(x-3) = 0$$

$$x = 0, 3$$

$$15 - \int_0^3 (x^3 - 3x^2 + 5)\,dx$$

The diagram shows the curve $y = 4 - x^2$ and the line $2x + y - 1 = 0$.

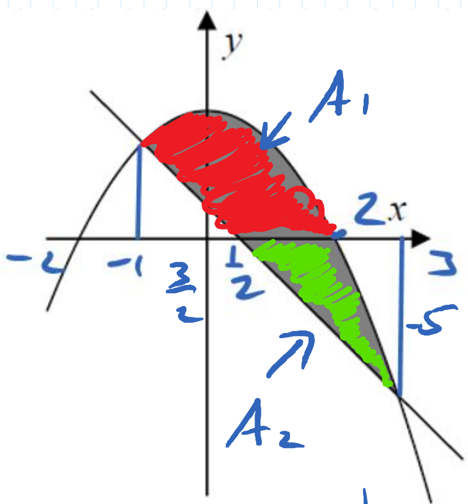
Calculate the area of the shaded region enclosed by the curve and the line.



$$4 - x^2 = 0$$
$$4 = x^2$$
$$x = \pm 2$$
$$y = 0 \quad x = \tfrac{1}{2}$$

$$2x + y - 1 = 0$$
$$y = 1 - 2x$$

$$1 - 2x = 4 - x^2$$
$$x^2 - 2x - 3 = 0$$
$$x = 3 \qquad x = -1$$



$x = -1$

$y = 3$

$$A_1 = \int_{-1}^{2} (4 - x^2)\,dx - \int_{-1}^{\frac{1}{2}} (1 - 2x)\,dx$$

$$\left[4x - \frac{x^3}{3}\right]_{-1}^{2} - \left[x - x^2\right]_{-1}^{\frac{1}{2}}$$

$$A_2 = \left| \int_{\frac{1}{2}}^{3} (1 - 2x)\,dx \int_{2}^{3} (4 - x^2)\,dx \right|$$