Special Case.

$g(x) = xe^x$ find $g'(x)$

hence find $\int xe^x \, dx$.

$$g(x) = xe^x$$

$$g'(x) = xe^x + e^x$$

$$\int g'(x)\,dx = g(x) \qquad \int xe^x \, dx$$

$$xe^x + e^x = g'(x)$$

$$xe^x = g'(x) - e^x$$

$$\int xe^x \, dx = \int (g'(x) - e^x) \, dx$$

$$= g(x) - e^x + c$$

$$= xe^x - e^x + c$$

$f(x) = x \ln x$ find $f'(x)$ hence

$\int_1^e \ln x \, dx$.

$$y = x \ln x$$

$$f'(x) = \frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x$$

$$f'(x) = 1 + \ln x$$

$$1 + \ln x = f'(x)$$

$$\ln x = f'(x) - 1$$

$$\int_1^e \ln x \, dx = f(x) - \left[x \right]_1^e$$

$$= \left[x \ln x - x \right]_1^e$$

$$= e \ln e - e - (1 \ln 1 - 1)$$

$$e - e + 1 = 1.$$

- $f(x) = 3x \sin 3x$ (i) Find $f'(x)$ (ii) Evaluate $\int_0^{\frac{\pi}{6}} 9x \cos 3x \, dx$

$$f(x) = 3x \sin 3x$$

$$f'(x) = 3x(3)\cos 3x + 3 \sin 3x$$

$$f'(x) = 9x \cos 3x + 3 \sin 3x$$

$$\int_0^{\frac{\pi}{6}} 9x \cos 3x \, dx = \int_0^{\frac{\pi}{6}} \left(f'(x) - 3\sin 3x \right) dx$$

$$= \left[3x\sin 3x + 3\left(\frac{1}{3}\right)\cos 3x \right]_0^{\frac{\pi}{6}}$$

$$= \left[3x \sin 3x + \cos 3x \right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{2} \sin \frac{\pi}{2} + \cos \frac{\pi}{2} - (0 + \cos 0)$$

$$\frac{\pi}{2} - 1$$