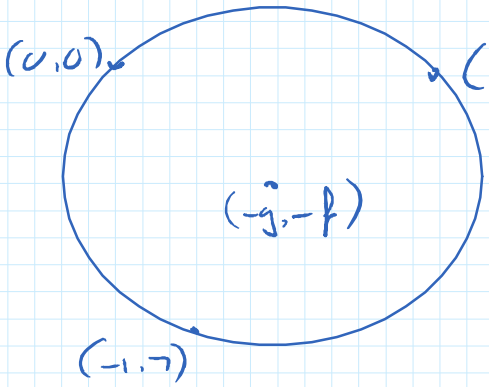
Circle where we cannot find centre.

Circle is $x^2+y^2+2gx+2fy+c=0$

Find circle which contains $(0,0)$, $(-2,4)$ and $(-1,7)$



$x^2+y^2+2gx+2fy+c=0$

Centre $(-g,-f)$

$r=\sqrt{g^2+f^2-c}$

$(0,0)$ $\quad c=0$

$(-2,4)$ $\quad 4+16-4g+8f=0$

$-4g+8f=-20$

$g-2f=5$

$(-1,7)$ $\quad 1+49-2g+14f=0$

$-2g+14f=-50$

$g-2f=5$

$-g+7f=-25$

$5f=20$

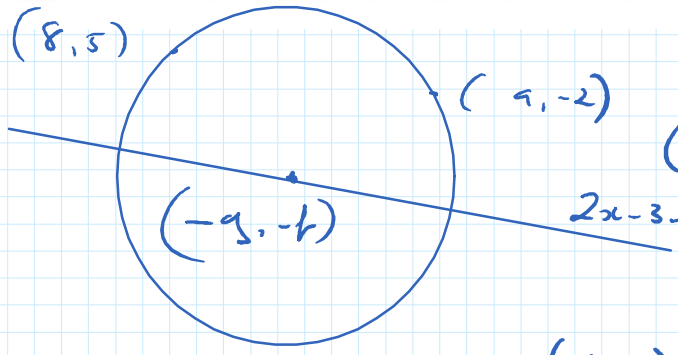
$f=4$

$g-8=5 \quad g=13$

$x^2+y^2+26x+8y=0$

A circle passes through the points $a(8,5)$ and $b(9,-2)$.
The centre of the circle lies on the line $2x-3y-7=0$.

(i) Find the equation of the circle.



$x^2+y^2+2gx+2fy+c=0$

$(9,-2)$: $81+4+18g-4f+c=0$

$18g-4f+c=-85$ -(i)

$(8,5)$: $64+25+16g+10f+c=0$

$16g+10f+c=-89$ -(ii)

Sum Eq between (i) & (ii)

$16g+10f+c=-89$

$-18g+4f+c=-85$

$-2g+14f=-4$

$g-7f=2$ -(iii)

$(-g,-f)$ into $2x-3y=7$

$-2g+3f=7$ -(iv)

$g-7f=2$ -(iii)

$-2g+3f=7$

$2g-14f=4$

$-11f=11$

$f=-1$

Sub into (iv) $g = -5$

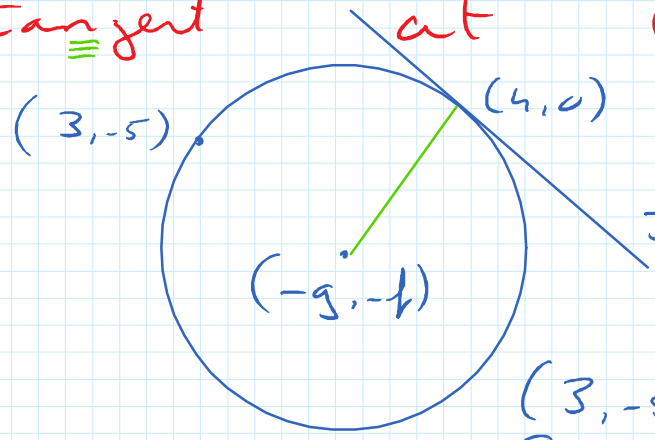
$16g + 10f + c = -89$

$-80 - 10 + c = -89$

$c = 1$

$x^2 + y^2 - 10x - 2g + 1 = 0$

Find circle through $(3, -5)$ which has $3x + 2y = 12$ as tangent at $(4, 0)$.



$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $(-g, -f)$

$r = \sqrt{g^2 + f^2 - c}$

$(3, -5)$

$9 + 25 + 6g - 10f + c = 0$

$6g - 10f + c = -34$

$(4, 0)$

$16 + 8g + c = 0$

$8g + 0f + c = -16$

$6g - 10f + c = -34$

$-8g + 0f + c = -16$

$-2g - 10f = -18$

$g + 5f = 9$

Green line

$2x - 3y = k$ $(4, 0)$

$2x - 3y = 8$ $(-g, -f)$

$-2g + 3f = 8$

$2(g+5f=9) \Rightarrow \quad 2g + 10f = 18$

$13f = 26$

$f = 2$

$g = -1$

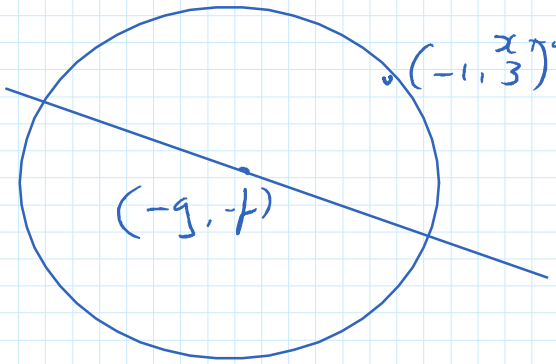
$8g + c = -16$

$-8 + c = -16 \quad c = -8$

$x^2 + y^2 - 2x + 4y - 8 = 0$

A circle of radius length $\sqrt{20}$ contains the point $(-1, 3)$. Its centre lies on the line $x + y = 0$.

Find the equations of the two circles that satisfy these conditions.



$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $(-g, -f)$

$r = \sqrt{20}$

$(-1, 3) \quad -2g + 6f + c = -10$

$(-g, -f) \quad x + y = 0$

$-g - f = 0$

$r = \sqrt{20}$

$\sqrt{g^2 + f^2 - c} = \sqrt{20}$

$g^2 + f^2 - c = 20$

$-g - f = 0 \Rightarrow -g = f$

$f = -g$

$$-2g + 6f + c = -10$$

$$-2g - 6g + c = -10$$

$$c = 8g - 10$$

$$g^2 + f^2 - c = 20$$

$$g^2 + g^2 - 8g + 10 - 20 = 0$$

$$2g^2 - 8g - 10 = 0$$

$$g^2 - 4g - 5 = 0$$

$$(g+1)(g-5) = 0$$

$$g = -1 \qquad g = 5$$

$$f = 1 \qquad f = -5$$

$c = 8g - 10$

$$c = -18 \qquad c = 30$$

$$x^2 + y^2 + 2x - 2y - 18 = 0$$

$$x^2 + y^2 + 10x - 10y + 30 = 0$$