Annuities

Investments at a constant over a period of time.

Investments I can make or borrowings that must be paid back in instalments.

Big ones are pensions and mortgages.

I will invest €200 every month at the start of a month into the bank at AER of 3%. Find value of investment at end of 3 years.

$3 \times 12 = 36$ instalements (Rough $36 \times 200 = €7200$)

$$(1+i)^{12} = 1.03$$

$$1+i = \sqrt[12]{1.03} = 1.002$$

$$F = P(1+i)^t \quad \text{Get money in future}$$

$$200(1.002)^{36} + 200(1.002)^{35} + \dots + 200(1.002)$$

$$200\left[1.002 + 1.002^2 + \dots\right]$$

$$a = 1.002 \qquad r = 1.002$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{1.002(1-1.002^{36})}{1-1.002}$$

$$= 37.69)$$

Ans $200(37.69) = €7538.16$

€6,000 is borrowed for 3 years at 8.5% APR. It is paid back in equal instalements at end of each month. Find the instalements.

When do I get to spend the money?

$$t = 3 \times 12 = 36$$

$$i = 0.085 \quad \text{APR} \Rightarrow \text{monthly.}$$

$$(1+i)^{12} = 1.085$$

$$1+i = \sqrt[12]{1.085}$$

$$= 1.0068$$

$$F = P(1+i)^t \quad \text{or} \quad P = \frac{F}{(1+i)^t} \checkmark$$

Lump $P = 6000$ $F = ??$ = instalment

$$P = \frac{F}{1.0068} + \frac{F}{1.0068^2} + \frac{F}{1.0068^3} - - -$$

$$= F\left[\frac{1}{1.0068} + \frac{1}{1.0068^2} + \dots\right]$$

$$a = \frac{1}{1.0068} \qquad r = \frac{1}{1.0068} \qquad n = 36$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{\frac{1}{1.0068}\left(1 - \left(\frac{1}{1.0068}\right)^{36}\right)}{1 - \frac{1}{1.0068}}$$

$$= 31.82$$

$$31.82F = 6000$$

$$F = €188.53$$

Check $188.53 \times 36 = €6787.18 > 6000$
because of interest.

I invest €x in bank at start of each month for 5 years. The AER is 2.5%. I get €3,542 at end of 5th year. Find x.

Lump Sum Instalements

$F = €3542$ $P = ??$

One is P other is F [Lump / Instale]

$$5 \times 12 = 60$$

$$(1+i)^{12} = 1.025$$

$$1+i = \sqrt[12]{1.025} = 1.002$$

$P(1+i)^t = F$ Start of month $P = P$

$$P(1.002)^{60} + P(1.002)^{59} + - - - + P(1.002) = 3542$$

Turn is around to make series easier

$P[1.002 + 1.002^2 + \dots -] = 3542$

$a = 1.002 \qquad r = 1.002 \qquad n = 60$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{1.002(1-1.002^{60})}{1-1.002}$$

$$= 63.92$$

$$63.92P = 3542$$

$$P = €55.40$$

$€55.40 \times 60 = 3324$ right cause of interest. gained on €55.40

Start of month

$= P(1.002)^{60} + P(1.002)^{59} + \dots P(1.002)$

cause 1st P is in for 60 month. Last is in for 1 month

End of month

$P(1.002)^{59} + P(1.002)^{58} + \dots + P$

1st is in for 59 month Last for 0 months.

$n = 60$

I borrowed a sum of money for 3 years at 8.6% APR. I pay €250 back each month at end of month. How much did I borrow?

$$t = 3 \times 12 = 36$$

$$(1+i)^{12} = 1.086$$

$$1+i = \sqrt[12]{1.086} = 1.0068$$

$$P = ?? \qquad F = 250$$

$$P = \frac{F}{(1+i)^t}$$

$$P = \frac{250}{1.0068} + \frac{250}{1.0068^2} + \cdots + \frac{250}{(1.0068)^{36}}$$

$$250\left[\frac{1}{1.0068} + \frac{1}{1.0068^2} + \cdots\right]$$

$$a = \frac{1}{1.0068} \qquad r = \frac{1}{1.0068} \qquad n = 36$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{\frac{1}{1.0068}\left(1 - \left(\frac{1}{1.0068}\right)^{36}\right)}{1 - \frac{1}{1.0068}}$$

$$= 31.78$$

Ans $250(31.78) = €7945.23$

If question start of month

$$P = 250 + \frac{250}{1.0068} + \cdots$$

$$250\left[1 + \frac{1}{1.0068}\right.$$

$$a = 1 \qquad r = \frac{1}{1.0068} \qquad n = 36.$$

Every 6 months $\Rightarrow$ $3 \times 2 = 6$

$$(1+i)^2 = 1.086$$