

Probability = chance

Prob of getting

(i) 6 on fair die?

(ii) tail on a coin?

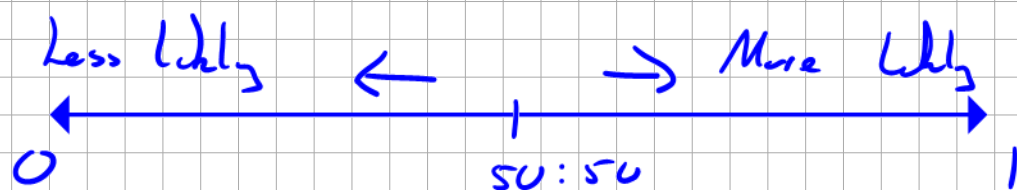
$$(i) P(6) = \frac{1}{6}$$

$$(ii) P(t) = \frac{1}{2}$$

$E$  = event

$$P(E) = \frac{\text{number of times } E \text{ happens}}{\text{total number of outcomes}}$$

Scale



Impossible

Even

Certain

AND = mult

OR = add.

2 coins are tossed. What is probability of getting a tail and a head.

$$T \quad T = \frac{1}{4}$$

$$H \quad H = \frac{1}{4}$$

$$H \quad T = \frac{1}{4}$$

$$T \quad H = \frac{1}{4}$$

$$2 \times 2 = 4 \text{ Total}$$

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

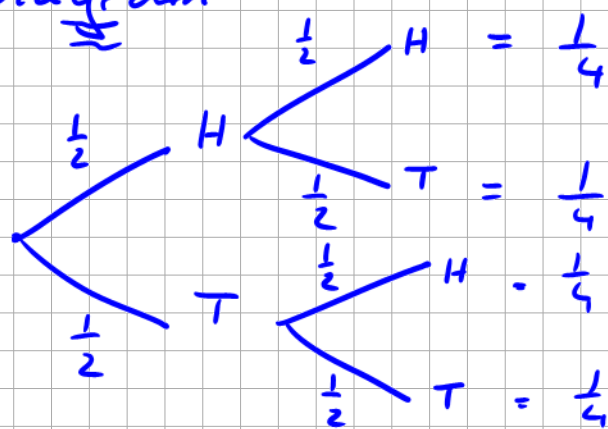
Sample space = write out all the answers.

ORDER IS VITAL.

Logic

$$\text{H and T or T and H} \\ \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{2}$$

Tree diagram



Marbles

5 blue and 7 green marbles in a bag. One is taken out and replaced. A second is taken out.

Use a tree diagram to find

prob

(i) both blue?

(ii) both same colour?

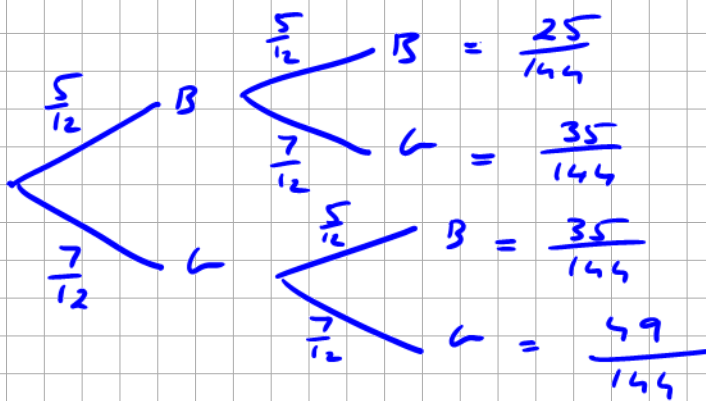
(iii) only one blue?

(iv) at least one blue?

5B

7G

Total 12.



$$(i) \frac{25}{144} = BB$$

$$(ii) BB \text{ or } GG$$

B and B or G and G

$$\frac{25}{144} + \frac{49}{144}$$

$$\frac{74}{144} = \frac{37}{72}$$

(iii) B and G or G and B

$$2 \left( \frac{35}{144} \right) = \frac{35}{72}$$

(iv) At least

BB or BG or GB

$$\frac{25}{144} + \frac{35}{144} + \frac{35}{144} = \frac{95}{144}$$

At  $1 - P(\text{both green})$

$$1 - \left( \frac{49}{144} \right) = \frac{95}{144}$$

5 pink and 9 green marbles in a bag. 2 are taken out simultaneously. Find using a tree diagram probability

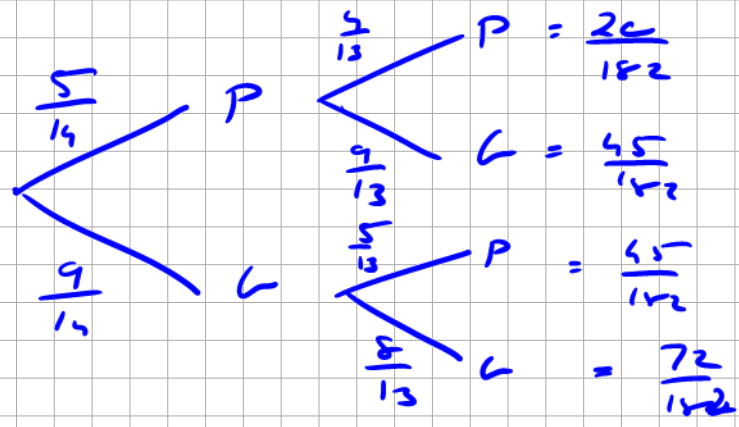
(i) both same colour?

(ii) at least one pink?

5P 9G Total 14

SP 4P  
9L 9L

SP SP  
9L 8L



(i)  $\frac{20}{182} + \frac{72}{182}$

(ii)  $1 - \frac{72}{182} = 1 - \frac{72}{182}$

### Sample Space

Write out all the answers.

Remember

- (i) Total is important
- (ii) Order is vital
- (iii)  $P(E) = \frac{\text{does}}{\text{total}}$ .

Two dice are thrown, results are noted and added. Find prob sum of results is 10 or more?

Total possible results =  $6 \times 6 = 36$

46 55 64  
56 65 66

$\frac{6}{36} = \frac{1}{36}$

1 1	2 1	3 1	4 1	5 1	6 1
1 2	2 2	3 2	4 2	5 2	6 2
1 3	2 3	3 3	4 3	5 3	6 3
1 4	2 4	3 4	4 4	5 4	6 4
1 5	2 5	3 5	4 5	5 5	6 5
1 6	2 6	3 6	4 6	5 6	6 6

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	5	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Question 1.  
 Read whole question.  
 Read first part - try  
 Stuck read the question.

A palindromic number is one that reads the same backwards as forwards, such as 727 or 38183.

- (i) This year, 2002, is a palindromic year. When is the next palindromic year?
- (ii) How many palindromic years are there from 1000 to 9999 inclusive?
- (iii) A whole number, greater than 9 and less than 10 000, is selected at random. What is the probability that the number is palindromic?

(i) 2112.

(ii)  $9 \times 10 \times 1 \times 1 = 90$

1001

2 3 4 5 6 7 8 9

1111

1000  $\rightarrow$  1100  $\rightarrow$  1

12

100  $\rightarrow$  1

13

$9999 - 1000 = \frac{8999}{100} = 89 + 1$

14

$10000 - 1000 = \frac{9000}{100} = 90$

15

16

17

18

19

$10 \times 9 = 90$

A palindromic number is one that reads the same backwards as forwards, such as 727 or 38183.

- (i) This year, 2002, is a palindromic year. When is the next palindromic year?
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Total possible  $10,000 - 9 = 9991$

$9999 - 9 = 9990$

2 digit     " , 22, 33     = 9

3 digit     101     2     3     =  $10 \times 9 = 90$

111

4 digit     =  $\frac{90}{189}$

$$\frac{189}{9990}$$

2 digit      $\frac{9 \times 1}{1} = 9$

3 digit      $\frac{9 \times 10 \times 1}{1} = 90$

4 digit     =  $\frac{90}{189}$

$$\frac{189}{9990}$$
~~$$\frac{189}{9989}$$~~

Eight cards are numbered 1 to 8. The cards numbered 1 and 2 are red, the cards numbered 3 and 4 are blue, the cards numbered 5 and 6 are yellow and the cards numbered 7 and 8 are black.

Four cards are selected at random from the eight cards.

Find the probability that the four cards selected are:

- (i) all of different colours
- (ii) two odd-numbered cards and two even-numbered cards
- (iii) all of different colours, two odd-numbered and two even-numbered.

Red  
1, 2

Blue  
3, 4

Yell  
5, 6

Black  
7, 8

$$R \quad B_e \quad Y \quad B_k$$

$$\frac{2}{8} \times \frac{2}{7} \times \frac{2}{6} \times \frac{2}{5} \times 4!$$

$$\frac{8 \times 6 \times 4 \times 2}{8 \times 7 \times 6 \times 5}$$

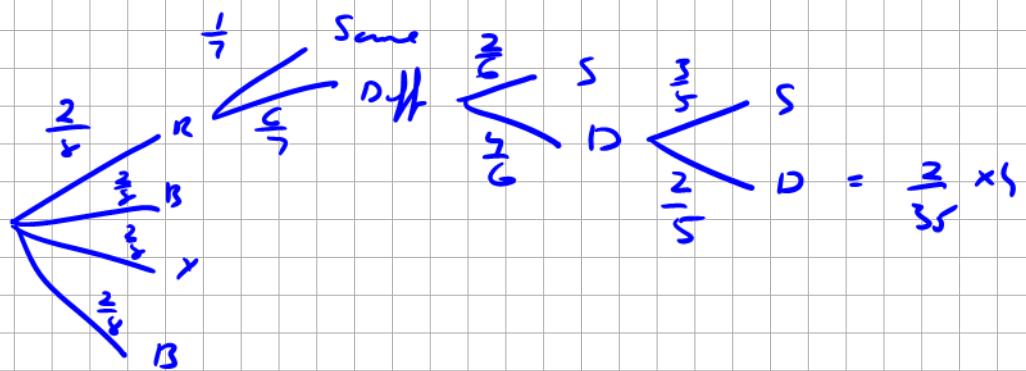
8 = any  
6 = left  
4 = left first 2.

order ↙

8 = all possible (use red)  
6 = only 6 left (Red used)

Select any

$$\frac{{}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{{}^8C_4} = \text{total}$$



Eight cards are numbered 1 to 8. The cards numbered 1 and 2 are red, the cards numbered 3 and 4 are blue, the cards numbered 5 and 6 are yellow and the cards numbered 7 and 8 are black.

Four cards are selected at random from the eight cards.

Find the probability that the four cards selected are:

- (i) all of different colours
- (ii) two odd-numbered cards and two even-numbered cards
- (iii) all of different colours, two odd-numbered and two even-numbered.

Total  ${}^8C_4$  Even 4 Odd 4

$$\frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}$$

OR

EE OO

$$\frac{4}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{3}{5} \times \frac{4!}{2!2!}$$

$\begin{matrix} \uparrow & \uparrow \\ E & O \end{matrix}$

Eight cards are numbered 1 to 8. The cards numbered 1 and 2 are red, the cards numbered 3 and 4 are blue, the cards numbered 5 and 6 are yellow and the cards numbered 7 and 8 are black.

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Find the probability that the four cards selected are:

- (i) all of different colours
- (ii) two odd-numbered cards and two even-numbered cards
- (iii) all of different colours, two odd-numbered and two even-numbered.

Red 1 2	BL 3 4	Ye 5 6	Bl 7 8
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Total  ${}^8C_4 = \frac{{}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{{}^8C_4}$

Ans (i)  $\times$  Ans (ii) = Right answer??

R 1 1 1	$\times$	Bl 3 4 4	$\times$	Y 5 5 6	$\times$	Bl 7 8 7	2 Odd 2 Even
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${}^8C_4 = 70$

Ans  $\frac{6}{70} = \frac{3}{35}$



A classroom contains 15 desks which are arranged in rows.

The front row contains 3 desks.

15 students are seated at random in the classroom, 8 of whom are boys and 7 of whom are girls.

Each desk seats only one student.

What is the probability that

- (i) three girls occupy the front row of desks?
- (ii) there are more boys than girls seated in the front row?
- (iii) there are two girls and one boy seated in the front row with the two girls seated next to each other?

(i)  $\frac{7}{15} \times \frac{6}{14} \times \frac{5}{13} = \frac{1}{13}$

$\frac{{}^7C_3}{{}^{15}C_3} = \frac{1}{13}$

8 B    7 G    Full 3 seats

(ii)  $\frac{{}^8C_2 \times {}^7C_1}{{}^{15}C_3} + \frac{{}^8C_3 \times {}^7C_0}{{}^{15}C_3}$

$\frac{8}{15} \times \frac{7}{14} \times \frac{7}{13} \times 3$

$\frac{8}{15} \times \frac{7}{14} \times \frac{6}{13}$

B B G    +    B B B

(iii)  $\frac{{}^7C_2 \times {}^8C_1}{{}^{15}C_3}$     Order ??

$2 \left( \frac{7}{15} \times \frac{6}{14} \times \frac{8}{13} \right) = \frac{16}{65}$

G G B    or    B G G

Five cards are drawn together at random from a standard pack of 52 playing cards. Find, in decimal form, correct to two significant figures, the probability that:

- (i) all five cards are diamonds
- (ii) all five cards are of the same suit
- (iii) the five cards are the ace, two, three, four and five of diamonds
- (iv) the five cards include the four aces.

(i)

$$\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48}$$

13 D                  P. J.                  5

$$\frac{{}^{13}C_5}{{}^{52}C_5} = 0.0004951$$

$$= 0.0005$$

(ii)

D or S or C or H

$$0.0004951 \times 4 = 0.00198$$

$$= 0.002$$

(iii)

A          2          3          4          5

$$\frac{1}{52} \times \frac{1}{51} \times \frac{1}{50} \times \frac{1}{49} \times \frac{1}{48} \times 5!$$

$$\frac{{}^5C_5}{{}^{52}C_5} = 0.000000384$$

$$0.00000038$$

$2.1 \times 10^{-3} \Rightarrow$  move decimal 3 places to left.

(iv)

4 A and 1 other

$$\frac{{}^4C_4 \times {}^{48}C_1}{{}^{52}C_5}$$

A A A A O

$$\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times \frac{48}{48} \times \frac{5!}{4!}$$

- Prob
- sample space
  - tree diagram
  - logic
  - combinations
  - permutations.

And = Mult

Or = Add

$$P(E) = \frac{\text{does}}{\text{Total}}$$

$$0 \leq P(E) \leq 1.$$

In a café there are 11 seats in a row at the counter.

Six people are seated at random at the counter.

How much more likely is it that all six are seated together than that no two of them are seated together?

No two

	Together		- no		two		together							
	<u>F</u>	<u>E</u>	<u>F</u>	<u>E</u>	<u>F</u>	<u>E</u>	—	—	—	—	—	—	—	—
	$\frac{6}{11}$	$\times \frac{5}{10}$	$\times \frac{5}{9}$	$\times \frac{4}{8}$	$\times \frac{4}{7}$	$\times \frac{3}{6}$	$\times \frac{3}{5}$	$\times \frac{2}{4}$	$\times \frac{2}{3}$	$\times \frac{1}{2}$	$\times \frac{1}{1}$			

Alternative  
Block.

$$\frac{6!5!}{11!}$$



Bubbles  $\rightarrow \frac{6!6!}{11!} \leftarrow$  group of 6.

$$\frac{6!6!}{11!} - \frac{6!5!}{11!} = \frac{5}{462}$$

Second Method

"C<sub>6</sub> = 462 = total ways of seats filled

Separate = 1 combinations.

Together = 6 combinations.

F E E E E E  
E F E E E E



A bag contains discs of three different colours.  
There are 5 red discs, 1 white disc and  $x$  black discs.  
Three discs are picked together at random.

- (i) Write down an expression in  $x$  for the probability that the three discs are all different in colour.
- (ii) If the probability that the three discs are all different in colour is equal to the probability that they are all black, find  $x$ .

	5R	1W	$x$ B	Total	$x+6$
RWB		R	W	B	
RBR					
WRB					
WBR					
BRW					
BRW					
	$\frac{5}{x+6}$	.	$\frac{1}{x+5}$	.	$\frac{x}{x+4}$
					. 3!
	B	B	B		
	$\frac{x}{x+6}$	.	$\frac{x-1}{x+5}$	.	$\frac{x-2}{x+4}$

$$\frac{30x}{(x+6)(x+5)(x+4)} = \frac{x(x-1)(x-2)}{(x+6)(x+5)(x+4)}$$

$$30x = x(x-1)(x-2)$$

$$30 = x^2 - 3x + 2$$

$$x^2 - 3x - 28 = 0$$

$$(x-7)(x+4) = 0$$

$$x = 7$$

$$x = -4$$

Discs

# Birthday Question.

3 people were asked 'which day of week were you born?'. Find prob

(i) all born on Monday?  
M and M and M

$$\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7} = \frac{1}{343}$$

(ii) all born on same day?

$$\text{Monday} \times 7 = \left(\frac{1}{7}\right)^3 \cdot 7 = \frac{1}{49}$$

Same Same Same

$$\frac{7}{7} \times \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$$

(iii) all born on different days?

~~$$\frac{1}{7} \times \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1}$$~~

$$\frac{7}{7} \times \frac{6}{7} \times \frac{5}{7} = \frac{30}{49}$$

(iv) 2 on same day?

$$\frac{7}{7} \times \frac{1}{7} \times \frac{6}{7} \times 3 = \frac{18}{49}$$

SSD  
SDS  
PSS

(v) at least 2 on same day?

2 on same or 3 on same day

$$\frac{18}{49} + \frac{1}{49} = \frac{19}{49}$$

$$\text{Alt} = 1 - P(\text{all diff}) = \frac{49}{49} - \frac{30}{49} = \frac{19}{49}$$

Lotto has 42 numbers with 6 winning numbers. What is prob I have

(i) 6 winning numbers?

(ii) 4 winning numbers?

6 W      36 L      Total 42

$$(i) \quad {}^{42}C_6 = 5,245,786$$

$$P(6W) = \frac{1}{5,245,786}$$

$$\begin{array}{cccccc} W & W & W & W & W & W \\ \frac{6}{42} & \times & \frac{5}{41} & \times & \frac{4}{40} & \times & \frac{3}{39} & \times & \frac{2}{38} & \times & \frac{1}{37} \end{array}$$

(ii) 4 W and 2 L

WWWWLL

$$\frac{6}{42} \times \frac{5}{41} \times \frac{4}{40} \times \frac{3}{39} \times \frac{36}{38} \times \frac{35}{37} \times \frac{6!}{4!2!}$$

$$\frac{{}^6C_4 \times {}^{36}C_2}{{}^{42}C_6}$$