Arithmetic Sequence and Series.

Info 1 . Define Arithmetic.

To get from one term to next add a fixed constant.

$U_n = 3n+1$ prove it is arithmetic

Arithmetic = Linear = gap is the same

$T_3 - T_2 = T_2 - T_1$ = idea behind arithmetic

$$\boxed{T_{n+1} - T_n = \text{constant.}}$$

Must use this to prove arithmetic

$U_n = 3n+1$

$U_{n+1} = 3(n+1)+1$

$= 3n+4$

$U_{n+1} - U_n = 3n+4-(3n+1)$

$= 3 = \text{constant} \Rightarrow \text{arithmetic}$

Remember $U_n = T_n$

d = common difference $\Rightarrow d = 3$

$$\boxed{\begin{array}{l} d = T_2 - T_1 \quad \text{given sequence} \\ d = T_{n+1} - T_n \end{array}}$$

$T_n = n^2 + 5n$ prove it is not arithmetic.

$T_n = n^2 + 5n$

$T_{n+1} = (n+1)^2 + 5(n+1)$

$= n^2 + 2n + 1 + 5n + 5$

$= n^2 + 7n + 6$

$T_{n+1} - T_n = n^2 + 7n + 6 - (n^2 + 5n)$

$= 2n + 6$ not a constant

not arithmetic

$S_n = n^2 + 7n$ prove arithmetic.

Defn $T_{n+1} - T_n = d$ = constant,

$T_n = S_n - S_{n-1}$

$S_{n-1} = (n-1)^2 + 7(n-1)$

$= n^2 - 2n + 1 + 7n - 7$

$= n^2 + 5n - 6$

$S_n - S_{n-1} = n^2 + 7n - (n^2 + 5n - 6)$

$T_n = 2n + 6$

$T_{n+1} = 2(n+1) + 6 = 2n + 8$

$T_{n+1} - T_n = 2n + 8 - (2n + 6) = 2$

2 = constant $\Rightarrow$ arithmetic

Note. $d \in \mathbb{R}$.

Info 2.

$a = T_1 = S_1$

$d = T_2 - T_1$ or $d = T_{n+1} - T_n$

3, 8, 13, 18 state a and d.

$a = 3$ $d = 5$

−7, −11, −15, a and d

$a = -7$ $d = -4$

In arithmetic sequence $T_n = 5 - 3n$ find a and d.

$a = T_1 = 2$

$T_2 = 5 - 6 = -1$

$d = T_2 - T_1 = -1 - 2 = -3$

EKKER

In an arithmetic series $S_n = 5n - n^2$ find a and d.

$a = S_1 = 5 - 1 = 4$

$d = T_2 - T_1$

$S_2 = T_1 + T_2$

$T_2 = S_2 - S_1$

$S_2 = 10 - 4 = 6$

$T_2 = 6 - 4 = 2.$

$d = T_2 - T_1 = 2 - 4 = -2.$

Alternative

$S_n = 5n - n^2$

$S_{n-1} = 5(n-1) - (n-1)^2$

$= 5n - 5 - (n^2 - 2n + 1)$

$= 5n - 5 - n^2 + 2n - 1$

$= -n^2 + 7n - 6$

$S_n - S_{n-1} = 5n - n^2 - (-n^2 + 7n - 6)$

$T_n = 6 - 2n$

$a = T_1 = 4$

$d = T_2 - T_1 \Rightarrow T_2 = 2$

$d = 2 - 4 = -2.$

$S_3 = T_1 + T_2 + T_3$

$S_2 = T_1 + T_2$

$S_1 = T_1$

$a = S_1 = T_1$

$S_2 - S_1 = \cancel{T_1} + T_2 - \cancel{T_1} = T_2$

1.b 3.

Do not need it is just idea.

a , $a+d$, $a+2d$, $a+3d$ $a+9d$
T_1 T_2 T_3 T_4 T_{10}

$T_n = a + (n-1)d$ = write it down but it is in the tables.

Find T_n of arithmetic sequence

(i) $\underset{T_1}{7}, \underset{T_2}{13}, \underset{T_3}{19}, ----$

$a = T_1$ $a = 7$ $d = 6$

$T_n = a + (n-1)d$

$T_n = 7 + 6(n-1)$

$= 7 + 6n - 6 = 6n + 1$

(ii) $3, -1, -5, -9$

$a = 3$ $d = -4$

$T_n = a + (n-1)d$

$T_n = 3 - 4(n-1)$

$= 3 - 4n + 4 = 7 - 4n$

$-8, -11, -14$ find T_{50}.

$a = -8$ $d = -3$ $n = 50 \Rightarrow n-1 = 49$

$T_n = a + (n-1)d$

$= -8 + 49(-3)$

$= -155$.

3 , 5 , 7 , 9 , -- which term has a value of 205.

$a = 3 \quad d = 2 \quad T_n = 205$

$T_n = a + (n-1)d = 205$

$3 + 2(n-1) = 205$

$n = 102.$ $\boxed{n \in N}$

Unknown term is always T_n.

$-5, -1, 3,$ has 411 as one term. Which term is it?

$a = -5 \quad d = 4 \quad T_n = 411$

$T_n = a + (n-1)d$

$= -5 + 4(n-1) = 411$

$-5 + 4n - 4 = 411$

$4n = 420$

$n = 105$

Info 4.

$$S_n = \sum_{n=1}^{n} T_n = \frac{n}{2}\{2a + (n-1)d\}.$$

Find S_n of $3 + 9 + 15 + \cdots$

$d = T_2 - T_1$
$= 9 - 3 = 6$

$a = 3 \quad d = 6$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{n}{2}\{2(3) + 6(n-1)\}$

$= \frac{n}{2}\{6n\} = 3n^2$

$T_n = 3-5n$ find S_n in an arithmetic sequence.

$a = T_1 = 3-5 = -2$

$d = T_2 - T_1 \Rightarrow T_2 = 3-5(2) = 3-10 = -7$

$d = -7-(-2) = -5$

$$S_n = \frac{n}{2}\left\{2a + (n-1)d\right\}$$

$$= \frac{n}{2}\left\{2(-2) - 5(n-1)\right\}$$

$$= \frac{n}{2}\left\{-4 - 5n + 5\right\}$$

$$= \frac{n}{2}\left(1-5n\right) = \frac{n-5n^2}{2}$$

Find $\sum_{n=1}^{n} (6n-3)$ hence $\sum_{n=1}^{50} (6n-3)$.

$T_n = 6n-3$

$T_1 = a = 6-3 = 3$

$T_2 = 6(2) - 3 = 9$

$d = T_2 - T_1 = 9-3 = 6$

$$S_n = \frac{n}{2}\left\{2a + (n-1)d\right\}$$

$$= \frac{n}{2}\left\{2(3) + 6(n-1)\right\} = 3n^2$$

$S_{50} = 3(50)^2 = 7500$

Find $4 + 11 + 18 + \dots + 130$

Arithmetic

$a = 4 \quad d = 7$

$$S_n = \frac{n}{2}\left\{2a + (n-1)d\right\}$$

$$= \frac{n}{2}\left\{2(4) + 7(n-1)\right\}.$$

What is the 130. Answer $= T_n$

$$T_n = 130$$
$$a + (n-1)d = 130$$
$$4 + 7(n-1) = 130$$
$$4 + 7n - 7 = 130$$
$$7n = 133$$
$$n = 19$$
$$S_{19} = \frac{19}{2}\left(8 + 7(8)\right) = 1{,}273$$

In arithmetic series $4 + 6 + 8 + \cdots$
for what n " $S_n = 460$

$$a = 4 \quad d = 2 \quad S_n = 460$$
$$S_n = \frac{n}{2}\left\{2a + (n-1)d\right\}$$
$$= \frac{n}{2}\left\{2(4) + 2(n-1)\right\} = 460$$
$$\Rightarrow n\left(4 + n - 1\right) = 460$$
$$n^2 + 3n = 460$$
$$n^2 + 3n - 460 = 0 \quad \text{LN } 460$$
$$n^2 + 23n - 20n - 460 = 0$$
$$n(n+23) - 20(n+23) = 0$$
$$(n+23)(n-20) = 0$$
$$\cancel{n = -23} \qquad n = 20$$
$$\text{Ans} \quad n = 20 \qquad \boxed{n \in \mathbb{N}}$$

Info 5:

Simultaneous Eqs

In an arithmetic progression $T_5 = 19$ and $T_8 = 27$ find a and d.

Note : Progression = sequence

$T_n = a + (n-1)d$

$T_5 = a + 4d = 19$

$T_8 = a + 7d = 27$

$3d = 8$

$d = \frac{8}{3}$

$a = \frac{25}{3}$

In an arithmetic sequence $T_4 = 15$ and $S_5 = 55$ find a and d

$T_4 = 15$ $S_5 = 55$ a and d.

$T_n = a + (n-1)d$

$T_4 = a + 3d = 15$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$S_5 = \frac{5}{2}\{2a + 4d\} = 55$

$5(a + 2d) = 55$

$-a + -2d = -11$

$a + 3d = 15$

$d = 4$

$a = 3.$

Info 6.

3 consecutive terms of an arithmetic progression are $x-1$, $3x+5$ and $2x-7$ find x.

$$x-1, \quad 3x+5, \quad 2x-7$$
$$T_1 \qquad T_2 \qquad T_3$$

$$T_2 - T_1 = T_3 - T_2$$

$$3x+5-(x-1) = 2x-7-(3x+5)$$
$$3x+5-x+1 = 2x-7-3x-5$$
$$2x+6 = -x-12$$
$$3x = -18$$
$$x = -6$$

$\boxed{x \in R}$

Info 7

Find 3 consecutive terms of an arithmetic sequence which add to 12 and multiply to 48.

$$x-d, \quad x, \quad x+d$$

Sum to 12 : $x-d+x+x+d = 12$
$$3x = 12$$
$$x = 4$$

Mult to 48 : $(4-d)(4)(4+d) = 48$ divide by 4

$4(4-d)(4+d) = 48$

$(4-d)(4+d) = 12$

$16-d^2 = 12$

$$16-d^2 = 12$$
$$-d^2 = -4$$
$$d^2 = 4$$
$$d = \pm 2$$

When $x = 4$ and $d = 2$ into $x-d$, x , $x+d$

2 , 4 , 6

When $x = 4$ and $d = -2$

6 , 4 , 2 .