Series.

Series = sum of terms of a sequence.

Find $1+2+3+4+5+6$

$1+2 = 3+3 = 6+4 = 10+5 = 15+6 = 21$

$S_6 = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 = \sum_{n=1}^{6} n$

$T_n = 2n+1$ find S_3

$T_1 = 2(1)+1 = 3$

$T_2 = 2(2)+1 = 5$

$T_3 = 2(3)+1 = 7$

$S_3 = T_1 + T_2 + T_3 = \sum_{n=1}^{3} (2n+1)$

$S_3 = 3+5+7 = 15$

Define S_n

$S_n = T_1 + T_2 + \cdots\cdots T_{n-1} + T_n = \sum_{n=1}^{n} T_n$

$S_4 = T_1 + T_2 + T_3 + T_4$

$S_3 = T_1 + T_2 + T_3$

$S_2 = T_1 + T_2$

$S_1 = T_1$

Given S_n to find T_n

$S_{12} = 101$ $\quad S_{13} = 99$ find T_{13}.

$$T_{13} = S_{13} - S_{12} = 99 - 101 = -2$$

$S_n = n^2 + 5n$ find T_n.

$$S_n = T_1 + T_2 + \cdots + T_{n-1} + T_n$$
$$S_{n-1} = T_1 + T_2 + \cdots + T_{n-1}$$

$$S_n - S_{n-1} = T_n$$

$$\boxed{T_n = S_n - S_{n-1}}$$

$$S_n = n^2 + 5n$$
$$S_{n-1} = (n-1)^2 + 5(n-1)$$
$$= n^2 - 2n + 1 + 5n - 5$$
$$= n^2 + 3n - 4$$
$$S_n - S_{n-1} = n^2 + 5n - (n^2 + 3n - 4)$$
$$= n^2 + 5n - n^2 - 3n + 4$$
$$= 2n + 4$$

$S_n = 7n - n^2$ find T_n.

$$S_n = 7n - n^2$$
$$S_{n-1} = 7(n-1) - (n-1)^2$$
$$= 7n - 7 - (n^2 - 2n + 1)$$
$$= 7n - 7 - n^2 + 2n - 1$$
$$= 9n - n^2 - 8 = -n^2 + 9n - 8$$
$$T_n = S_n - S_{n-1}$$
$$= 7n - n^2 - (-n^2 + 9n - 8)$$

$$= 7n - n^2 + n^2 - 9n + 8$$

$$= 8 - 2n$$