Solve $x^2 - 2x - 8 = 0$

$x^2 - 2x - 8 = 0$ GN -8

$x^2 - 4x + 2x - 8 = 0$ Sub -2

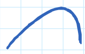
$x(x-4) + 2(x-4) = 0$

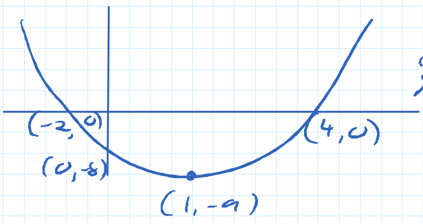
$(x-4)(x+2) = 0$ Factors.

$x - 4 = 0$ $x + 2 = 0$

$x = 4$ $x = -2$ Roots

$y = x^2 - 2x - 8$

$+x^2$  $-x^2$ 



$y = x^2 - 2x - 8$

Minimum $x = \frac{-2+4}{2}$

$x = 1$

$y = 1 - 2 - 8$

$= -9$

$(1, -9)$

$y = 4x - x^2$. Roots, maximum point and a rough graph.

$4x - x^2 = 0$ Roots $\Rightarrow y = 0$

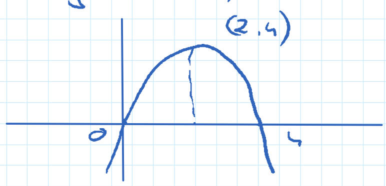
$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ $x = 4$

Maximum $x = \frac{0+4}{2} = 2$

$y = 4(2) - 2^2 = 4$ $(2,4)$



Complete the square.

Simplify (i) $(x+3)^2 = x^2 + 6x + 9$

(ii) $(x+6)^2 = x^2 + 12x + 36$

(iii) $(x-10)^2 = x^2 - 20x + 100$

(iv) $(x+4)^2 = x^2 + 8x + 16$

Complete the square on

$$y = x^2 - 8x + 20$$
$$y = x^2 - 8x + 16 + 20 - 16$$
$$y = (x-4)^2 + 4$$

$(x-4)^2$ smallest value of

-8 no since a square

$(x-4)^2 = 0$

Minimum $(4,4)$

$$y = x^2 - 2x - 8$$
$$y = x^2 - 2x + 1 - 8 - 1$$
$$y = (x-1)^2 - 9$$

Minimum $(1,-9)$

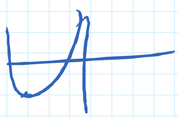
Complete the square and find minimum point on

$$y = x^2 + 6x - 1$$
$$y = x^2 + 6x + 9 - 1 - 9$$
$$y = (x+3)^2 - 10$$

$x + 3 = 0$ $\quad$ $y = (-3+3)^2 - 10$

$x = -3$

Minimum $(-3, -10)$



2 Real roots.

$$y = x^2 - 8x + 17.$$
$$y = x^2 - 8x + 16 + 17 - 16$$
$$y = (x-4)^2 + 1$$

$x - 4 = 0$

$x = 4$

Minimum $(4,1)$

$(x-4)^2 \geq 0$

No real roots.

Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

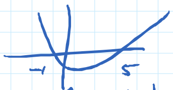
$x^2 - 4x - 5 = 0$

$\frac{\quad \sqrt{\quad}}{}$

$\frac{4 \pm \sqrt{36}}{2}$

$\frac{4+6}{2}$, $\frac{4-6}{2}$

$x = 5$ $\quad x = -1$



2 real distinct roots

$(x+5)(x+1)$

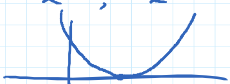
$x^2 - 4x + 4 = 0$

$\frac{4 \pm \sqrt{\quad}}{}$

$\frac{4 \pm \sqrt{0}}{2}$

$\frac{4+0}{2}$, $\frac{4-0}{2}$

2 , 2



2 real equal roots

$(x-2)(x-2)$

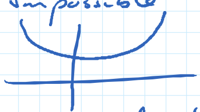
$x^2 - 4x + 5 = 0$

$\frac{4 \pm \sqrt{16-20}}{2}$

$\frac{4 \pm \sqrt{-4}}{2}$

$\sqrt{-4} \neq \sqrt{4}$

Impossible



No real roots
Imaginary roots
Complex roots

Learn

$b^2 - 4ac > 0 \Rightarrow$ 2 real different roots

$b^2 - 4ac = 0 \Rightarrow$ 2 real equal roots
perfect squares

$b^2 - 4ac < 0 \Rightarrow$ no real roots
imaginary roots
complex

$b^2 - 4ac \geq 0 \Rightarrow$ real roots

For what K does $x^2 + Kx + 6 = 0$ have 2 equal roots.

$$b^2 - 4ac = 0$$

$$K^2 - 4(1)(6) = 0$$

$$K^2 = 24$$

$$K = \pm\sqrt{24}$$

Hard Quadratics

Solve $\quad (3x-1)^2 - 2(3x-1) - 8 = 0$

Solve

$$x^2 - 3x = 10$$

$$x(x-3) = 10$$

$x = 10 \qquad x - 3 = 1$

$$x = 4$$

$(3x-1)^2 - 2\cancel{(3x-1)} = 8$

$\cancel{(3x-1)^2} - (3x-1) = 10$

$(3x-1)^2 - 2(3x-1) - 8 = 0$

Let $t = 3x-1$

$t^2 - 2t - 8 = 0$

$(t-4)(t+2) = 0$

$t = 4 \qquad t = -2$

$3x - 1 = 4 \qquad 3x - 1 = -2$

$3x = 5 \qquad 3x = -1$

$x = \frac{5}{3} \qquad x = -\frac{1}{3}$

Find minimum point of

$y = x^2 - 10x - 3$.

$y = x^2 - 10x + 25 - 3 - 25$

$y = (x-5)^2 - 28$

Minimum point $\Rightarrow$ lowest value of y

$y = (x-5)^2 - 28$ = perfect square

$(x-5)^2 = 0$ lowest

$x - 5 = 0$

$x = 5$

$y = -28$

Minimum $(5, -28)$

Minimum point of

$y = x^2 - 12x + 5$.

$y = x^2 - 12x + 36 + 5 - 36$

$y = (x-6)^2 - 31$

$x - 6 = 0$

$x = 6$

$(6, -31)$ Minimum

Find minimum point of

$y = x^2 - 14x + 5$

$y = x^2 - 14x + 5$

$y = x^2 - 14x + 49 + 5 - 49$

$y = (x-7)^2 - 44$

Minimum $(7, -44)$

$y = x^2 - 7x + 1$

$y = x^2 - 7x + \frac{49}{4} + \frac{4}{4} - \frac{49}{4}$

$y = \left(x - \frac{7}{2}\right)^2 - \frac{45}{4}$ $\qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Minimum $\left(\frac{7}{2}, -\frac{45}{4}\right)$

$y = -x^2 + 9x + 1$ $\qquad y = -(x^2 - 9x - 1)$

$-y = x^2 - 9x - 1$

$-y = x^2 - 9x + \frac{81}{4} - \frac{4}{4} - \frac{81}{4}$

$-y = \left(x - \frac{9}{2}\right)^2 - \frac{85}{4}$

$y = -\left(x - \frac{9}{2}\right)^2 + \frac{85}{4}$

Maximum $\left(\frac{9}{2}, \frac{85}{4}\right)$

$+x^2$ $\cup$ $\qquad -x^2$ $\cap$

$y = 5x - x^2$

$-y = x^2 - 5x + \frac{25}{4} - \frac{25}{4}$

$-y = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4}$

$$y = \frac{25}{4} - \left(x - \frac{5}{2}\right)^2$$

Maximum $\left(\frac{5}{2}, \frac{25}{4}\right)$

$$y = x^2 - 6x + 4$$

$$y = x^2 - 6x + 9 + 4 - 9$$

$$= (x-3)^2 - 5$$

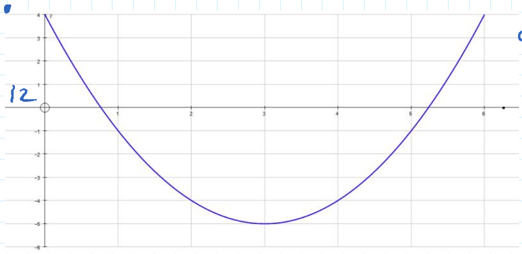
Minimum $(3, -5)$

$(x-3)^2 - 5 = 0$

$(x-3)^2 = 5$

$x - 3 = \pm\sqrt{5}$

$x = 3 \pm \sqrt{5}$

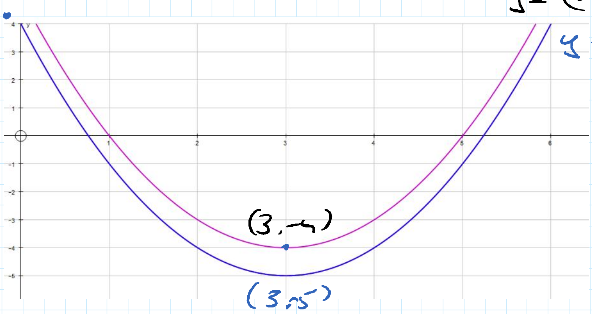


$y = x^2 - 6x + 4$.

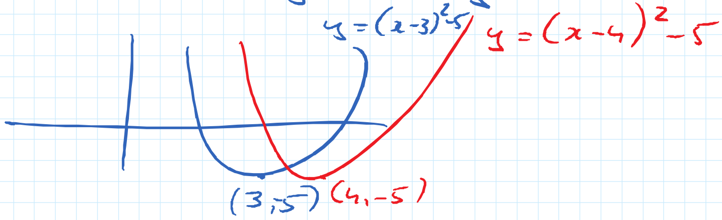
Whole graph to move up 1.

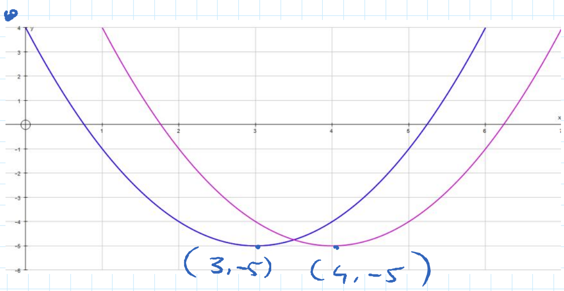
$y = (x-3)^2 - 4$

$y = (x-3)^2 - 5$



Move to right by 1





Move graph left 2 and up 1.

$y = (x-3)^2 - 5$

Minimum $(3, -5)$

New Minimum $(1, -4)$

$y = (x-1)^2 - 4$

Find maximum point of

$y = 8 - 6x - x^2$

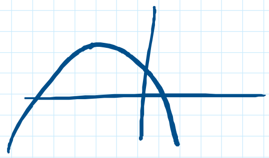
$y = -(x^2 + 6x - 8)$

$y = -(x^2 + 6x + 9 - 8 - 9)$

$y = -((x+3)^2 - 17)$

$y = 17 - (x+3)^2$

Maximum $(-3, 17)$



$y = 2x^2 - 3x + 7$. Find minimum point.

$y = 2(x^2 - \frac{3}{2}x + \frac{7}{2})$

$\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$y = 2(x^2 - \frac{3}{2}x + \frac{9}{16} + \frac{56}{16} - \frac{9}{16})$

$y = 2((x - \frac{3}{4})^2 + \frac{47}{16})$

$\cancel{2}\left(\frac{47}{\cancel{16}\ 8}\right)$

$= 2(x - \frac{3}{4})^2 + \frac{47}{8}$

Minimum $\left(\frac{3}{4}, \frac{47}{8}\right)$

$$y = 2x^2 - 7x - 9$$

$+x^2 \ \cup$

$-x^2 \ \cap$

$$y = 2\left(x^2 - \frac{7}{2}x - \frac{9}{2}\right)$$

$$= 2\left(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{72}{16} - \frac{49}{16}\right)$$

$$= 2\left(\left(x - \frac{7}{4}\right)^2 - \frac{121}{16}\right)$$

$$\left(\frac{7}{4}\right)^2 = \frac{49}{16}$$

$$= 2\left(x - \frac{7}{4}\right)^2 - \frac{121}{8}$$

Minimum $\left(\frac{7}{4}, -\frac{121}{8}\right)$