Add

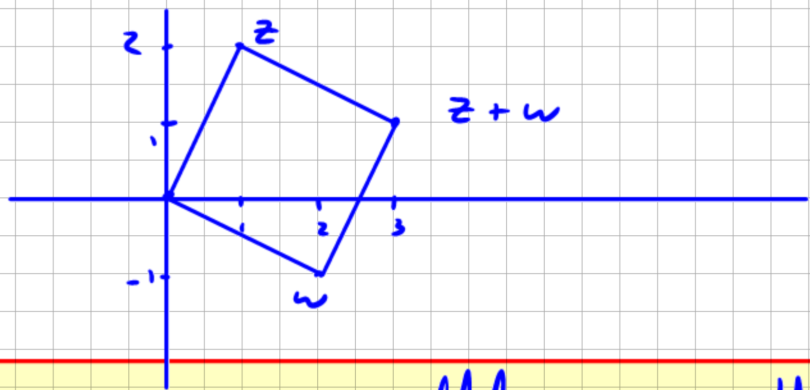
$z = 1+2i$ $w = 2-i$ show on Argand diagram z, w and $z+w$.

$z+w = 1+2i+2-i$

$= 3+i$

Rule Add real to real

Imaginary to imaginary



$\boxed{z+w \text{ is parallelogram with } 0, z \text{ and } w.}$

$z = 2+5i$ $w = 3-4i$ Find

(i) $z+w$

(ii) $z-w$

(iii) $2z+3w$

(i) $2+5i+3-4i = 5+i$

(ii) $2+5i-(3-4i) = -1+9i$

(iii) $2(2+5i)+3(3-4i)$

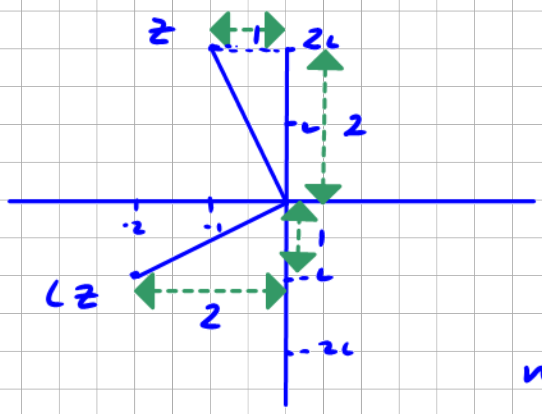
$4+10i+9-12i = 13-2i$

Multiply by i

On Argand diagram plot

$z = -1 + 2i$ and iz

$z = -1 + 2i$



iz

$= i(-1 + 2i)$

$= -i + 2i^2$

$= -2 - i$

$\boxed{i^2 = -1}$

$\boxed{iz = 90^\circ \text{ anti-clockwise rotation.}}$

m of 0 to z $= -\frac{2}{1} = \frac{\text{rise}}{\text{run}}$

m of 0 to iz $= \frac{1}{2}$

Multiplication.

Simplify

$\boxed{i^2 = -1}$

(i) $(x+3)(x+5)$

$x^2 + 5x + 3x + 15$

$x^2 + 8x + 15$

(ii) $(2+3i)(4+2i)$

$8 + 4i + 12i + 6i^2$ $\qquad i^2 = -1$

$2 + 16i$

(iii) $(5-4i)^2$

$25 - 40i + 16i^2$

$9 - 40i$

(iv) $(5+3i)(5-3i)$

$25 - 9i^2$

$34 \in \mathbb{R}.$

Surds $a + \sqrt{b}$ = compound surd

$a - \sqrt{b}$ = surd conjugate

$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b \in R$. means the sq root is gone.

Complex Conjugate

$z = a + bi$

a = Real part of z = $Re(z)$

b = Imaginary part of z = $Im(z)$

Every complex number has a complex conjugate $\bar{z} = a - bi$.

Change sign of imaginary part.

Axial symmetry in x-axis. $\Rightarrow$ directly in line one above the x-axis and one below.

$(a + bi)(a - bi) = a^2 - b^2 i^2$

$= a^2 + b^2 \in R \Rightarrow$ no imaginary part.

$z = -2 + 3i$ find $\bar{z}$ and $z\bar{z}$.

Show on Argand diagram

(i) z (ii) $\bar{z}$ (iii) $Re(z)$ (iv) $Im(z)$

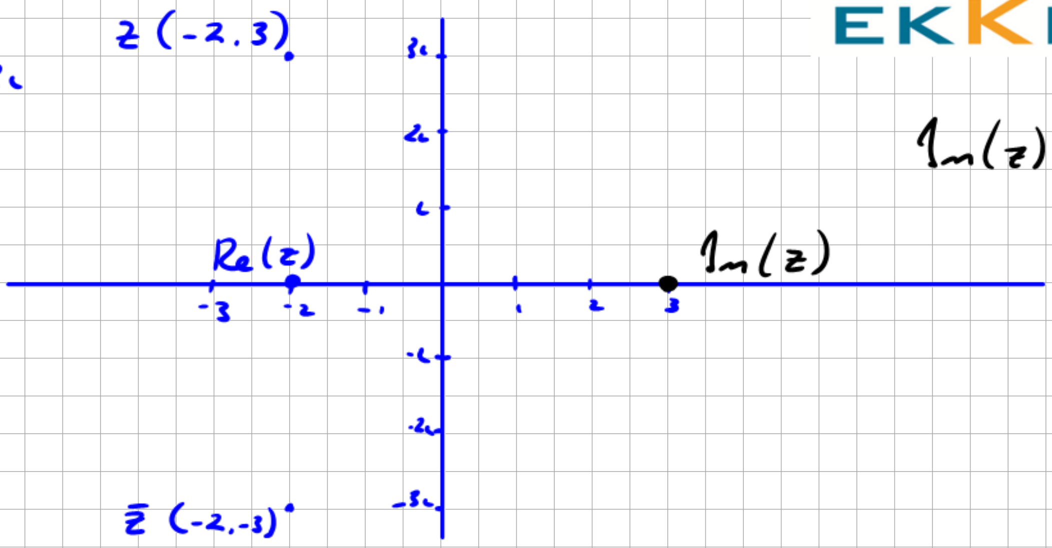
$z = -2 + 3i$

$\bar{z} = -2 - 3i$

$z\bar{z} = (-2 + 3i)(-2 - 3i)$

$4 - 9i^2 = 13$ $i^2 = -1$

$z = -2 + 3i$



$\operatorname{Im}(z) = 3$

Divide

Simplify $\dfrac{2+3i}{4+5i}$

Rule: Multiply above and below by conjugate of bottom.

$$\frac{2+3i}{4+5i} \cdot \frac{4-5i}{4-5i}$$

Top $(2+3i)(4-5i)$

$8 - 10i + 12i - 15i^2 = 23 + 2i$

Bottom $(4+5i)(4-5i) = 16 - 25i^2 = 41$

$$\frac{23+2i}{41} = \frac{23}{41} + \frac{2}{41}i$$

$z = 3 - 5i$ simplify $\dfrac{z}{\bar{z}}$

$$\frac{z}{\bar{z}} = \frac{3-5i}{3+5i} \cdot \frac{3-5i}{3-5i}$$

Top $9 - 15i - 15i + 25i^2 = -16 - 30i$

Bottom $9 - 25i^2 = 34$

$$\frac{-16-30i}{34} = \frac{-8-15i}{17}$$

Simplify $\left(\frac{-2+3i}{3+2i}\right)^9$

Using BOMDAS

$$\frac{-2+3i}{3+2i} \cdot \frac{3-2i}{3-2i}$$

Top $\quad -6+4i+9i-6i^2 = 13i$

Bottom $\quad 9-4i^2 = 13$

$$\left(\frac{-2+3i}{3+2i}\right)^9 = i^9 = i^2 \cdot i^2 \cdot i^2 \cdot i^2 \cdot i$$
$$= (i^2)^4 i$$
$$= (-1)^4 i = (1)i = i$$

Simplify (i) i^3

Need i^2. Use rules of indices

$$a^p \cdot a^q = a^{p+q}$$
$$(a^p)^q = a^{pq}$$
$$i^3 = i^2 \cdot i = -i$$

(ii) i^4.

$$(i^2)^2 = (-1)^2 = 1$$

$(-1)^n = +1$ when n is even

$(-1)^n = -1$ when n is odd

(iii) i^{99}

$$(i^2)^{49} \cdot i = (-1)^{49} i = -i$$

(iv) i^{221}

$$(i^2)^{110} \cdot i$$
$$= i$$

EKKER

Simplify

$$\left(\frac{1}{2+i}+\frac{2}{3-i}\right)^{100}$$

$$\frac{1}{2+i}+\frac{2}{3-i}=\frac{3-i+2(2+i)}{(2+i)(3-i)}$$

$$=\frac{3-i+4+2i}{6-2i+3i-i^2}$$

$$=\frac{7+i}{7+i}=1$$

$$1^{100}=1$$

Prove

$$\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$$

$$z_1=a+bi \qquad z_2=x+yi$$

$$z_1+z_2=a+x+i(b+y)$$

$$\overline{z_1+z_2}=a+x-i(b+y)$$

$$\overline{z_1}=a-bi$$

$$\overline{z_2}=x-yi$$

$$\overline{z_1}+\overline{z_2}=a+x-i(b+y)$$

QED