Combinations.

Order is NOT important.

Combinations = choice.

How many 2 letter combinations are possible from A, B, C, D each letter used only once?

AB BC CD ABCD
AC BD
AD

Ans 6

Permutation 4×3 → combination $\frac{4 \times 3}{2!} = 6$

How many 3 letter combinations using A, B, C, D each letter used only once?

ABC BCD
ABD
ACD

$\frac{4 \times 3 \times 2}{3!} = \frac{{}^4P_3}{3!}$

${}^4C_3 = 4$

n = total number of possible event

r = number chosen.

$n, r \in N$

EKKER

$${}^nC_r = \frac{{}^nP_r}{r!} = \frac{\frac{n!}{(n-r)!}}{\frac{r!}{1}}$$

$$\boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

${}^nC_2 = 10$ find n.

$${}^nC_2 = \frac{n!}{2!(n-2)!} = 10$$

$$\Rightarrow \frac{n(n-1)(n-2)!}{2(n-2)!} = 10$$

$$\Rightarrow n^2 - n = 20$$

$$n^2 - n - 20 = 0$$

$$(n-5)(n+4) = 0$$

$$n = 5 \qquad \cancel{n = -4}$$

$2!(n-2)! \neq (2n-4)!$ BOMDAS

How many ways can 3 people be chosen from 6?

$n = 6 \qquad r = 3$

$${}^nC_r = {}^6C_3 = 20$$

How many ways can team of 11 be chosen from panel of 18?

$$^{18}C_{11} = 31,824$$

A lotto has 24 numbers pick 4 to win. How many combinations are possible?

$$^{24}C_4 = 10,626$$

Exam 6 questions do

(i) 4

(ii) 5 how many combinations.

(i) $^6C_4 = 15$

(ii) $^6C_5 = 6$

Two events.

5 men and 3 women apply for 2 jobs. How many ways can jobs be filled? 5M 3W Total 8 Choose 2.

~~$8 \times 7 = 56$~~ $^8C_2 = 28$

How many ways of

(i) one man and one woman are employed.

5 M 3 W Total 8

1 M AND 1 W

$${}^5C_1 \times {}^3C_1 = 15$$

(ii) at least one woman?

5 M 3 W 2 Jobs.

English → 1 W and 1 M

or

2 W and 0 M

$${}^3C_1 \times {}^5C_1 = 3 \times 5 = 15$$

$${}^3C_2 \times {}^5C_0 = 3 \times 1 = 3$$

$$18$$

Alternative = Total − 2 men.

Note $${}^nC_0 = {}^nC_n = 1$$

$${}^nC_1 = n$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$${}^8C_2 = \frac{8!}{2!\,6!}$$

$${}^8C_6 = \frac{8!}{6!\,2!}$$

$${}^9C_3 = {}^9C_6$$

$${}^3C_2 = {}^3C_1 = 3$$

$${}^nC_r = {}^nC_{n-r}$$

Pascal's Triangle. (Leave alone)

$(x+y)^0 = 1$

$(x+y)^1 = 1x + 1y$

$(x+y)^2 = 1x^2 + 2xy + 1y^2$

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$

$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$\binom{1}{0}$ $\binom{5}{1}$ $\binom{10}{2}$ $\binom{10}{3}$ $\binom{5}{4}$ $\binom{1}{5}$

Expand $(x+y)^5$

$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Find first 3 terms of $(x+y)^9$

$\binom{9}{0}x^9 + \binom{9}{1}x^8y + \binom{9}{2}x^7y^2$

$x^9 + 9x^8y + 36x^7y^2$

First 3 terms of $(x-y)^{10} = (x+(-y))^{10}$

$\binom{10}{0}x^{10} + \binom{10}{1}x^9(-y) + \binom{10}{2}x^8(-y)^2$

A test has 6 questions in section A and 8 in B. How many ways can I answer 5 questions at least 2 from each section.

6A 8B Total 14

3A and 2B ${}^6C_3 \times {}^8C_2 = 560$

or +

2A and 3B ${}^6C_2 \times {}^8C_3 = 840$

1400

~~2A and 2B and 1A or 1B~~

~~${}^6C_2 \times {}^8C_2 \times ({}^4C_1 + {}^6C_1)$~~

5 teacher 6 students apply for 3 places on a council. How many ways can place be filled with more students than teacher?

5 T 6 S Total 11 Fills

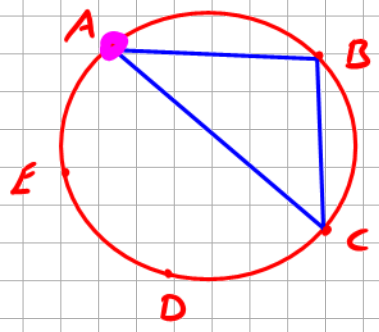
2 S and 1 T $= {}^6C_2 \times {}^5C_1$

or +

3 S and 0 T ${}^6C_3 \times {}^5C_0$

$= 95$

Diagrams.



How many triangles can be formed?

How if

(i) A is always used?

(ii) A is never used?

ABC
BAC

~~$5 \times 4 \times 3 = 60$~~ $^5C_3 = 10$ no A

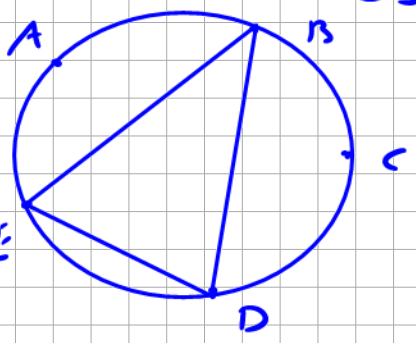
(i) $^4C_2 = 6$ $^5C_3 - {}^4C_3$

Total - no A.

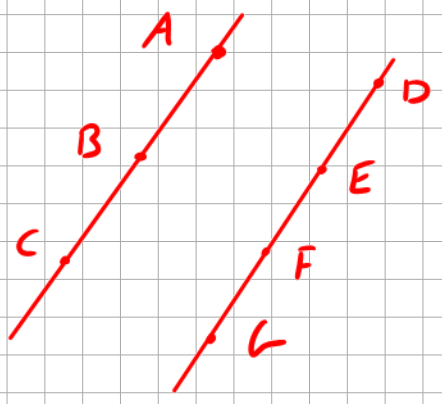
A used means 4 left pick any 2.

(ii) A not used means 4 left pick 3

$^4C_3 = 4$.



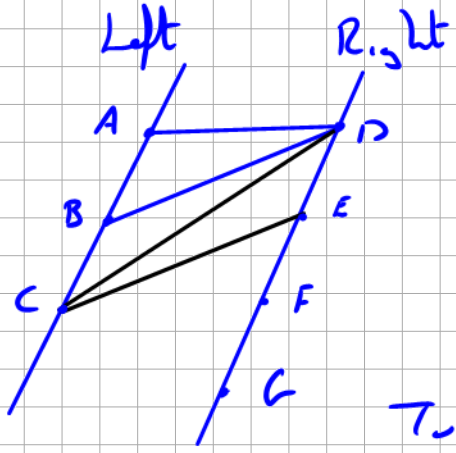
BCD



Parallel lines. Gaps are equal between points. Find number of

(i) triangles

(ii) parallelograms which may be formed.



$^7C_3$ ($\frac{2}{5}$ marks)

ABD
ABE → sample space

Blue △

Total	Left 3	Right 4	
Blue	Left 2	Right 1	
	$^3C_2 \times {}^4C_1$	=	12
Blue	Left 1	Right 2	
	$^3C_1 \times {}^4C_2$	=	18

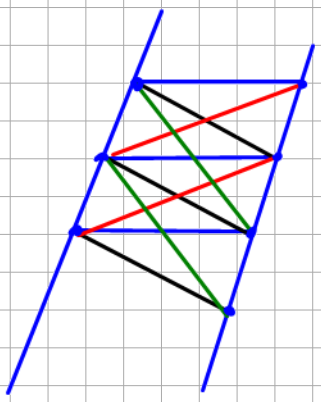
$\frac{18}{30}$

Alternative

Total $^7C_3$ − straight lines

35 − 5 = 30

ABC
DEF
DEL
DFL
EFG.



$^3C_2 \times {}^4C_2$

3 + 3 + 1 + 1 = 8

Thick question.

A panel of 12 is used for a team of 5. How many teams can be formed?

How many if A will not play when B plays?

EKKER

$^{12}C_5 = 792$

A not plays B plays = $^{10}C_4$

A plays B not plays = $^{10}C_4$

Neither plays = $^{10}C_5$

672.

Alternative Total − both

$^{12}C_5 - {}^{10}C_3 = 672$

Alt A not plays 11 p.l 5

$^{11}C_5$ =

or B plays 10 p.l 4.

$^{10}C_4$ =