Centre (-g,-f)

Simplify

$$(x-h)^2+(y-k)^2=r^2$$

$$x^2-2hx+h^2+y^2-2ky+k^2-r^2=0$$

$$x^2+y^2-2hx-2ky+h^2+k^2-r^2=0$$

Centre $(-g,-f)$

$$x^2+y^2+2gx+2fy+c=0$$

$$c=g^2+f^2-r^2$$

$$r^2=g^2+f^2-c$$

$$r=\sqrt{g^2+f^2-c}$$

Circle of form $x^2+y^2+2gx+2fy+c=0$ has centre $(-g,-f)$ and $r=\sqrt{g^2+f^2-c}$

Find centre and radius of

$$x^2+y^2+4x-6y-7=0$$

Centre $(-2,3)$ $\quad -g \quad -f$

$r=\sqrt{g^2+f^2-c}$ $\quad r=\sqrt{2^2+(-3)^2+7}$

$$=\sqrt{4+9+7}$$

$$=\sqrt{20}$$

(ii) $\quad x^2+y^2-6x+8y-1=0$

Centre $(3,-4)$

$$r=\sqrt{g^2+f^2-c}$$

$$\sqrt{9+16+1}=\sqrt{26}$$

Find centre and radius of

(i) $x^2+y^2-6x+8y-5=0$

Of form $x^2+y^2+2gx+2fy+c=0$

(ii) $x^2+y^2-6x+8y-1=0$

Centre $(3,-4)$

$r=\sqrt{g^2+f^2-c}$

$\sqrt{9+16+1}=\sqrt{26}$

Find centre and radius of

(i) $x^2+y^2-6x+8y-5=0$

Of form $x^2+y^2+2gx+2fy+c=0$

(iii) $x^2+y^2-7x+3y-5=0$

$r=\sqrt{g^2+f^2-c}$

Centre $\left(\frac{7}{2},-\frac{3}{2}\right)$

$r=\sqrt{\frac{49}{4}+\frac{9}{4}+\frac{20}{4}}$

$=\frac{78}{4}=\frac{\sqrt{78}}{2}$

(iv) $x^2+y^2-3x-1=0$

Centre $\left(\frac{3}{2},0\right)$

$r=\sqrt{\frac{9}{4}+\frac{4}{4}}$

$= \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}$

$(3,-1)$ is on $x^2+y^2-2x+3y+k=0$

Find the radius.

$3^2+(-1)^2-2(3)+3(-1)+k=0$

$9+1-6-3+k=0$

$k=-1$

Centre $\left(1,-\frac{3}{2}\right)$

$r=\sqrt{1+\frac{9}{4}+1}=\sqrt{\frac{17}{4}}$

$(2,-5)$ is on $x^2+y^2-6x+8y+k=0$.

Find k.

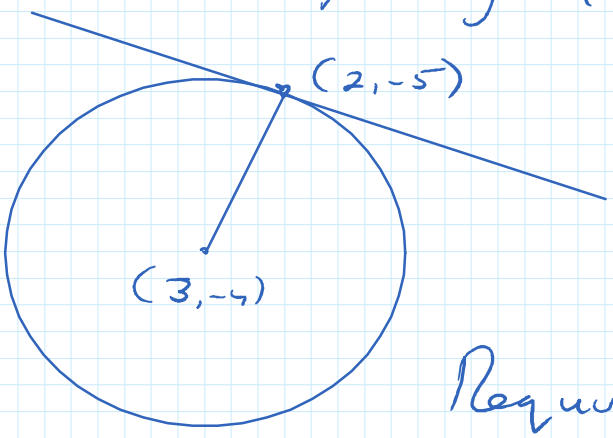
Find tangent at $(2,-5)$

$2^2+(-5)^2-6(2)+8(-5)+k=0$

$k=23$

Centre $(3,-4)$

$r=\sqrt{9+16-23}=\sqrt{2}$



$(2,-5)$ $(3,-4)$
$x_1\ y_1$ $x_2\ y_2$

$m=\frac{-4+5}{3-2}=1$

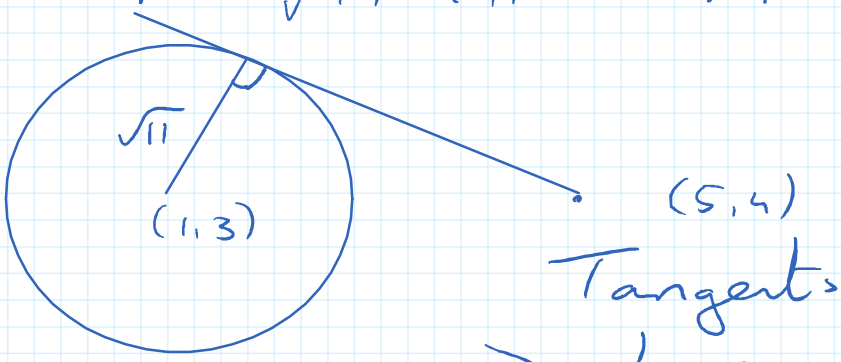
Required $m=-1$

$y+5=-1(x-2)$

Find tangents from $(5,4)$ to $x^2+y^2-2x-6y-1=0$.

Centre $(-g,-f)$ Centre $(1,3)$

$r=\sqrt{g^2+f^2-c}$ $\quad r=\sqrt{1+9+1}=\sqrt{11}$



Tangents $\Rightarrow$ lines

$y-y_1=m(x-x_1)$

$y-4=m(x-5)$

$y-4=mx-5m$

$mx-y+4-5m=0$ to $(1,3)$

$a=m \quad b=-1 \quad c=4-5m \quad x_1=1 \quad y_1=3$

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$\frac{|m-3+4-5m|}{\sqrt{m^2+1}}=\sqrt{11}$$

$$|-4m+1|=\sqrt{11}\sqrt{m^2+1}$$

$$16m^2-8m+1=11(m^2+1)$$

$$16m^2-8m+1=11m^2+11$$

$$5m^2-8m-10=0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$ Finish later (*)

Find where $x+y=6$ and $x^2+y^2+2x-4y-20=0$ intersect.

$$x+y=6$$
$$y=6-x$$
$$x^2+(6-x)^2+2x-4(6-x)-20=0$$
$$x^2+36-12x+x^2+2x-24+4x-20=0$$
$$2x^2-6x-8=0$$
$$x^2-3x-4=0$$
$$(x-4)(x+1)=0$$
$$x=4 \quad x=-1$$

or.

$$x=6-y$$
$$(6-y)^2+y^2+2(6-y)-4y-20=0$$
$$36-12y+y^2+y^2+12-2y-4y-20=0$$
$$2y^2-18y+28=0$$
$$y^2-9y+14=0$$
$$(y-2)(y-7)=0$$
$$y=2 \quad y=7$$

Find where $x^2+y^2-4x-6y-12=0$ cuts x-axis.

x-axis $\quad y=0$

$$x^2-4x-12=0$$
$$(x-6)(x+2)=0$$
$$x=6 \qquad x=-2$$

Is $(5,-2)$ inside, on or outside $x^2+y^2-4x+6y-3=0$

Centre $(2,-3)$

$$r=\sqrt{4+9+3}=\sqrt{16}=4$$

Sub in points

$$25+4-20-12-3$$
$$-6<0 \Rightarrow \text{inside.}$$

Find centre and radius of $x^2+y^2-10x-6y-3=0$. Is $(2,1)$ on, inside or outside the circle.

$$x^2+y^2+2gx+2fy+c=0$$

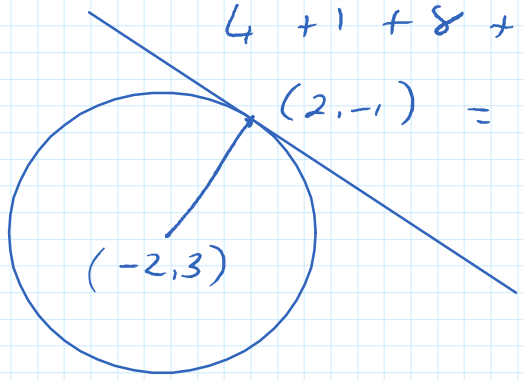
Centre $(-g,-f)$ $\quad r=\sqrt{g^2+f^2-c}$

Centre $(5,3)$ $\quad r=\sqrt{25+9+3}$

$$=\sqrt{37}$$

$$2^2+1^2-20-6-3<0 \Rightarrow \text{inside.}$$

$(2,-1)$ is on $x^2+y^2+4x-6y+k=0$.
Find k. Find tangent at $(2,-1)$.



$4+1+8+6+k=0$

$k=-19$

$x^2+y^2+4x-6y-19=0$

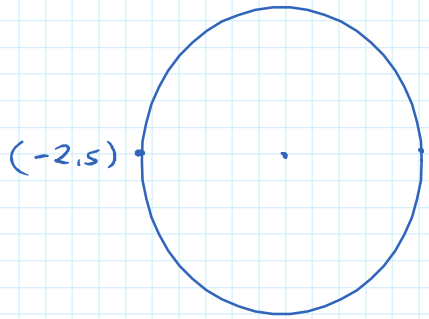
Centre $(-2,3)$

$(2,-1)$ $(-2,3)$

$m=\frac{3+1}{-2-2}=-1$

$y+1=-1(x-2)$

Find circle with $(-2,5)$ and $(6,3)$ as end points of a diameter.



Centre = midpoint.

$\left(\frac{2}{h},\frac{4}{k}\right)$

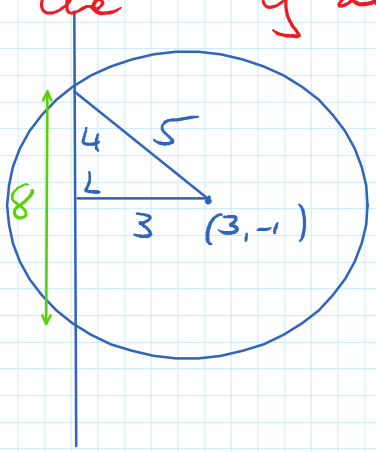
Radius = distance from $(2,4)$ to $(6,3)$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{4^2+(-1)^2}=\sqrt{17}$

$(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-4)^2=17$

Find circle centre (3, -1) which cuts a chord of length 8 units on the y-axis.



$(x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y+1)^2=r^2$

$(x-3)^2+(y+1)^2=25$