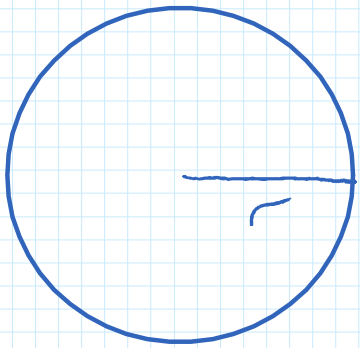
Shapes - rates of change.

$\frac{dr}{dt} = m/s$

$\frac{dA}{dt} = m^2/s$

$\frac{dV}{dt} = m^3/s$

A circle is increasing at rate of $4m^2/s$. Find rate of increase of radius.



Got $\frac{dA}{dt} = 4$

Need $\frac{dr}{dt}$

$A = \pi r^2$

Found $\frac{dA}{dr} = 2\pi r$

$\frac{dr}{dt} = \frac{dA}{dt} \div \frac{dA}{dr} = \frac{4}{2\pi r} = \frac{2}{\pi r}\ m/s$

$\frac{dA}{dt} \times \frac{dr}{dA}$

A circle is increasing at rate of 3m/s find rate of increase of area when $r = 7m$.

$$\frac{dr}{dt} = 3 \qquad \frac{dA}{dt} = ??$$

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

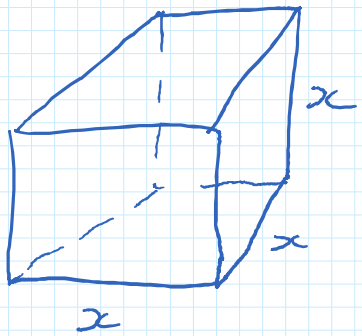
$$\frac{dA}{dt} = \frac{dr}{dt} \cdot \frac{dA}{dr} = 3(2\pi r)$$

$$r = 7 \quad \frac{dA}{dt} = 42\pi \ m^2/s$$

The sides of a cube are changing at rate of 6m/s. Find rate of increase of

(i) area.

(ii) volume.



$$\frac{dx}{dt} = 6$$

Need $\frac{dA}{dt}$

$$A = 6x^2$$

$$\frac{dA}{dx} = 12x$$

$$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt} = 72x \text{ m/s}$$

$$V = x^3$$

$$\frac{dV}{dx} = 3x^2 \qquad \frac{dx}{dt} = 6$$

$$\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt} = 18x^2 \text{ m}^3/\text{s}$$

$$\text{m/s} = \frac{d\,\text{length}}{dt}$$

$$\text{m}^2/\text{s} = \frac{dA}{dt}$$

$$\text{m}^3/\text{s} = \frac{dV}{dt}$$

The radius of a sphere is increasing at rate of 6m/s. Find rate of increase of volume.

Got $= \frac{dr}{dt} = 6$ Need $= \frac{dV}{dt}$

Formulae $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dr}{dt} \cdot \frac{dV}{dr} = 6(4\pi r^2)$

$= 24\pi r^2 \left(m^3/s\right)$

A sphere has an area increasing at rate of $6m^2/s$. Find rate of increase of radius.

Got $\frac{dA}{dt} = 6$ Need $\frac{dr}{dt}$

Find $A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$\frac{dr}{dt} = \frac{dA}{dt} \div \frac{dA}{dr} = \frac{6}{8\pi r}$ m/s

A sphere has a volume increasing at rate of $12\,m^3/s$.
Find rate of increase of radius when $r = 3\,m$.

Know $\frac{dV}{dt} = 12$ Need $\frac{dr}{dt} = ?$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr} = \frac{12}{4\pi r^2}\ m/s$$

$r = 3$ $\frac{12}{36\pi}\ m/s$

$\frac{1}{3\pi}\ m/s$

A sphere has an area increasing at rate of $7\,m^2/s$.
Find rate of increase of volume.

Know $\frac{dA}{dt} = 7$ Need $\frac{dV}{dt}$

$A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

$\frac{dA}{dr} = 8\pi r$ $\frac{dV}{dr} = 4\pi r^2$

$$\frac{dV}{dt} = \frac{dV}{dr} \div \frac{dA}{dr} \times \frac{dA}{dt} = \frac{dV}{\cancel{dr}} \cdot \frac{\cancel{dr}}{\cancel{dA}} \cdot \frac{dA}{dt}$$

$$= \frac{4\pi r^2}{8\pi r} \cdot 7 = \frac{7r}{2} \ m^3/s.$$

A sphere has a volume increasing at rate of $6m^3/s$. Find rate of increase of area when $r = 3m$.

Got $\frac{dV}{dt} = 6$ Need $\frac{dA}{dt} = ??$

$$V = \frac{4}{3}\pi r^3 \qquad A = 4\pi r^2$$

$$\frac{dV}{dr} = 4\pi r^2 \qquad \frac{dA}{dr} = 8\pi r$$

$$\frac{dA}{dt} = \frac{dV}{dt} \div \frac{dV}{dr} \times \frac{dA}{dr} = \frac{6}{4\pi r^2} \cdot 8\pi r$$

$$= \frac{12}{r}$$

$r = 3 \quad \frac{dA}{dt} = 4 m^2/s$

A cylinder has height 3 times the radius. The radius is changing at rate of 5m/s. Find rate of increase of volume?

$\frac{dr}{dt} = 5$ Need $\frac{dV}{dt}$

$V = \pi r^2 h \qquad h = 3r$

$$V = \frac{3}{4}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = \left(4\pi r^2\right) m^3/s .$$