Logs are the inverse to indices.

Indice $b^p = n$

b = base

p = power

n = number.

$$\log_b n = p$$

$$2^x = 1.5 \quad = \quad \log_2 1.5 = x$$

$$b^p = n \quad \Leftrightarrow \quad \log_b n = p$$

$2^x = 5$ find x.

$$\log_2 5 = x$$

$$x = 2.3$$

$$3^x = 10$$

$$\log_3 10 = x$$

Compound continuous formulae

$$F = Pe^{rt}$$

P = principal = start with, borrow

$F = A$ = future value or amount

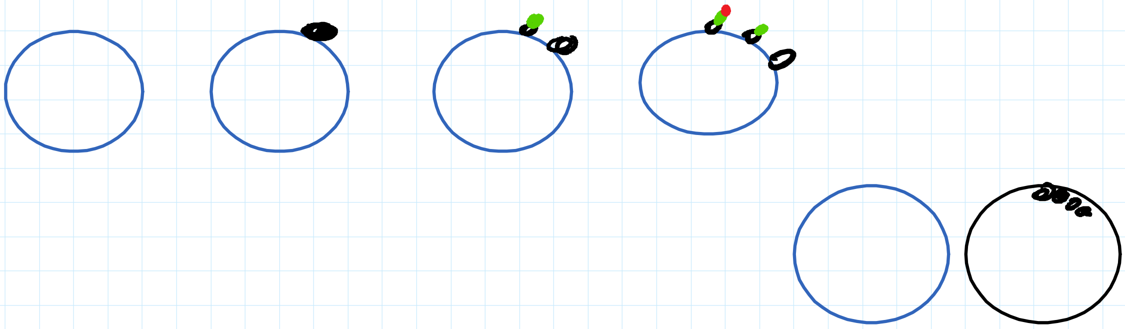
= end with

r = rate of interest as a DECIMAL

t = time.

$e = 2.71$

$€1 \rightarrow 100\% \rightarrow €2.71$



$$F = Pe^{rt}$$

How much will €500 amount to in 3 years at 4% compounded continuously?

$P = 500 \quad t = 3 \quad r = 0.04$

$$F = Pe^{rt}$$
$$= 500e^{3(0.04)}$$
$$= 500e^{0.12}$$
$$= €563.748$$
$$= €563.75$$

Compounded annually

$$F = P(1+i)^t$$
$$= 500(1.04)^3$$

€6000 is worth €6500 at the end of 3 years of investment at r%. Find r to 1 decimal place?

$P = 6000 \quad F = 6500 \quad t = 3 \quad r = ?$

$$F = Pe^{rt}$$

$$6500 = 6000e^{3r}$$

$$\frac{6500}{6000} = e^{3r}$$

$$\log_e \frac{6500}{6000} = 3r$$

$$\ln \frac{6500}{6000} = 3r$$

$$r = 0.0266$$
$$= 2.66\%$$
$$= 2.7\%$$

$\log_e = \ln$

How long to nearest year will it take €3,000 to amount to €3,500 at 2% AER?

$$F = Pe^{rt}$$

$P = 3000 \quad F = 3500 \quad r = 0.02$

$$3500 = 3000e^{0.02t}$$

$$\frac{3500}{3000} = e^{0.02t}$$

$$\ln \frac{3500}{3000} = 0.02t$$

$$t = 7.7$$

$$t = 8 \text{ years}$$

If $t = 7.001$ Answer 8 years.

Bacteria B is given by $B = 500e^{rt}$. Find

(i) Initial bacteria levels?

(ii) It doubles in 3 hours find r?

$$B = 500e^{rt}$$

Initial $\Rightarrow$ $t = 0$

$$B = 500e^{0} \Rightarrow 500$$

$t = 3$ $B = 1000$ $r = ??$

$$1000 = 500e^{3r}$$

$$2 = e^{3r}$$

$$\ln 2 = 3r$$

$$r = 23\%$$

Note AER = annual equivalent rate - saving

APR = annual percentage rate = borrow

Number of rats R is given by $R = 60e^{rt}$. Find

(i) initial population.

(ii) population after 2 years at rate of growth of 5%.

(iii) how long for population to reach 30,000 when $r = 5\%$.

$$R = 60e^{rt}$$

(i) $t = 0$ $\quad R = 60$

(ii) $r = 0.05$ $\quad t = 2$

$$R = 60e^{0.1}$$
$$= 66.3 = 66$$

(iii) $R = 30000$ $\quad r = 0.05$

$$30000 = 60e^{0.05t}$$
$$500 = e^{0.05t}$$
$$\ln 500 = e^{0.05t}$$
$$t = 124.29$$
$$= 125 \text{ days}.$$

€4,000 amounts to €4650 after 4 years at r% AER. Find r to 1 decimal place.

$$F = Pe^{rt}.$$

$P = 4000 \qquad F = 4650 \qquad t = 4 \qquad r = r$

$$4650 = 4000\,e^{4r}$$

$$\frac{4650}{4000} = e^{4r}$$

$$\log_e \frac{4650}{4000} = 4r$$

$$r = 0.0376$$

$$= 3.8\%$$

A tractor cost €95,000.
After 4 years it is worth €48,000.
Given $F = Pe^{rt}$

(i) state possible values of r for depreciation (decline)

(ii) find r to 1 decimal place.

(i) $r < 0$

(ii) $P = 95000 \qquad F = 48000 \qquad t = 4$

$$F = Pe^{rt}$$

$$48000 = 95000\,e^{4r}$$

$$\frac{48000}{95000} = e^{4r}$$

$$\ln\left(\frac{48}{95}\right) = 4r$$

$$r = -0.1706$$

$$= -17.1\%$$

$F = Pe^{rt}$

Learn

(i) Initial = start $\Rightarrow$ $t = 0$

(ii) Growth $\Rightarrow$ $r > 0$

Decline $\Rightarrow$ $r < 0$

Rules of Logs

$a^m \cdot a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$a^1 = a$

$a^0 = 1$

$\log a + \log b = \log ab$

$\log a - \log b = \log \frac{a}{b}$

$\log_a a = 1$

$\log_a 1 = 0$

$\log a^n = n \log a$

$$\log a + \log b = \log ab$$

$$2\log a = \log a + \log a = \log a^2$$

$$3\log a = \log a + \log a + \log a = \log a^3$$

$$4\log a = \log a^4$$

Type 1:

$$\log 2 + \log x = \log 8$$

$$\log 2x = \log 8$$

$$2x = 8$$

$$x = 4.$$

Goal : 1 log = 1 log

$$\log x - \log 3 = \log 5.$$

$$\log \frac{x}{3} = \log 5$$

$$\frac{x}{3} = 5$$

$$x = 15$$

$$\log x = \log_{10} x$$

Solve $\quad \log 2 + \log x = \log 50$

$$1 \log = 1 \log$$

$$\log 2x = \log 50$$

$$2x = 50$$

$$x = 25$$

$$\log x - \log 7 = \log 3$$

$$\log \frac{x}{7} = \log 3$$

$$\frac{x}{7} = 3$$

$\log a - \log b = \log \frac{a}{b}$

$n \log a = \log a^n$

$$x = 21$$

$$2 \log x - \log 3 = \log 12.$$

$$\log x^2 - \log 3 = \log 12$$

$$\log \frac{x^2}{3} = \log 12$$

$2\log x = \log x + \log x$
$= \log x^2$

$\frac{x^2}{3} = 12$

$x^2 = 36$

~~$x = 6$~~ $\quad x = \pm 6$

$x = 6$

$\log_3 27 = t \qquad \log_3(-27) = t$

$3^t = 27 \qquad 3^t = -27$

$t = 3 \qquad$ Impossible

Log(negative) = impossible

$\log 36 \neq 2\log 18$

$\log_{10} 100 = t \qquad \log_{10} 200 = t$

$10^t = 100 \qquad 10^t = 200$

$t = 2$

Solve $\quad \log x + \log(x-2) = \log 8$

$\log x(x-2) = \log 8$

$x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4 \qquad \cancel{x = -2}$

$\log_2 4 = \log_2 8$

$$\log_2 4 = x \qquad \log_2 8 = y$$

$$2^x = 4 \qquad 2^y = 8$$

$$x = 2 \qquad y = 3$$

$$\log 2x - \log(x-7) = \log 3.$$

$$\log_x \frac{2x}{x-7} = \log 3$$

$$\frac{2x}{x-7} = 3$$

$$2x = 3(x-7)$$

$$x = 21$$

$$\log x + \log 30 = \log 90$$

$$\log 30x = \log 90$$

$$30x = 90$$

$$x = 3$$

$$\log 5x - \log(x-7) = \log 4$$

$$\log \frac{5x}{x+7} = \log 4$$

$$\frac{5x}{x+7} = 4$$

$$5x = 4x + 28$$

$$x = 28$$

Solve $\quad 2\log x - \log(x+4) = \log 2.$

$$\log x^2 - \log(x+4) = \log 2$$

$$\log \frac{x^2}{x+4} = \log 2$$

$$\frac{x^2}{x+4} = 2$$

$$x^2 = 2(x+4)$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4 \qquad x = -2$$

Answer $x = 4.$

Solve $\log_2(x-2) + \log_2 x = 3$

$$\log_2 x(x-2) = 3$$

$$\log_2(x^2-2x) = 3 \qquad \log_b n = p$$

$$x^2 - 2x = 2^3 \qquad b^p = n$$

$$x^2 - 2x = 8$$

$$x^2 - 2x - 8 = 0$$

$$x = 4 \qquad \cancel{x = -2}$$

$$\log_3 x - \log_3(x-1) = 2$$

$$\log_3 \frac{x}{x-1} = 2$$

$$\frac{x}{x-1} = 3^2$$

$$x = 9x - 9$$

$$9 = 8x$$

$$\frac{9}{8} = x$$

$$\log(x^2+6) - \log(x^2-1) = 1$$

$$\log_{10} \frac{x^2+6}{x^2-1} = 1$$

$$\frac{x^2+6}{x^2-1} = 10^1$$

$$x^2+6 = 10x^2-10$$

$$9x^2 = 16$$

$$x^2 = \frac{16}{9}$$

$$x = \pm\frac{4}{3}$$

$$x = \frac{4}{3}$$

Change of base

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Change $\log_{25} x$ to base 5.

$$\log_{25} x = \frac{\log_5 x}{\log_5 25}$$

$$\log_5 25 = p$$
$$5^p = 25$$

$$= \frac{\log_5 x}{2}$$

Change $\log_x 36$ to base 6.

$$\log_x 36 = \frac{\log_6 36}{\log_6 x}$$

$$\log_6 36 = t$$
$$6^t = 36$$

$$= \frac{2}{\log_6 x}$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Change $\log_x 8$ to base 2.

$$\frac{\log_2 8}{\log_2 x} = \frac{3}{\log_2 x}$$

$\log_2 8 = t$

$2^t = 8$

$2^t = 2^3$

$t = 3$

$\log_x 27$ to base 3.

$$\log_x 27 = \frac{\log_3 27}{\log_3 x} = \frac{3}{\log_3 x}$$

Solve

$$\log_4(3x+1) = \log_2(x-1)$$

$$\log_4(3x+1) = \frac{\log_2(3x+1)}{\log_2 4}$$

$$= \frac{\log_2(3x+1)}{2}$$

$$\frac{\log_2(3x+1)}{2} = \log_2(x-1)$$

$$\log_2(3x+1) = 2\log_2(x-1)$$

$$\log_2(3x+1) = \log_2(x-1)^2$$

$$3x+1 = x^2 - 2x + 1$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$\cancel{x = 0} \qquad x = 5$$

Solve $\log_8 x - \log_9 x = 1$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$\log_9 x = \frac{\log_8 x}{\log_8 9}$$

$\log_8 9 = t$

$8^t = 9$

$$\log_8 x - \frac{\log_8 x}{1.056} = 1$$

$$1.056 \log_8 x - \log_8 x = 1.056$$

$$0.056 \log_8 x = 1.056$$

$$\log_8 x = \frac{1.056}{0.056}$$

$$\log_8 x = 1.97$$

$$8^{1.97} = x$$

$$x = 61.$$

Solve

$$4\log_x 2 - \log_2 x - 3 = 0$$

Change $\log_2 x$ to base x

$$\log_2 x = \frac{\log_x x}{\log_x 2} = \frac{1}{\log_x 2}$$

$$4\log_x 2 - \frac{1}{\log_x 2} - 3 = 0$$

$$4(\log_x 2)^2 - 1 - 3\log_x 2 = 0$$

$$t = \log_x 2$$

$$4t^2 - 3t - 1 = 0$$

$$(4t+1)(t-1) = 0$$

$$t = -\frac{1}{4} \qquad t = 1$$

$$\log_x 2 = -\frac{1}{4} \qquad \log_x 2 = 1$$

$$x^1 = 2$$

$$(x^{-\frac{1}{4}})^{-4} = 2^{-4}$$

$$x^1 = 2^{-4}$$

$$x^{-\frac{1}{4}} = 2$$

$$x^{-1} = 2^4$$

$$x = 2^{-4}$$

$$= \frac{1}{2^4} = \frac{1}{16}$$

$a = \log 2$ $\quad b = \log 3$ write in terms of a and/or b

(i) $\log 6$

$$a = \log 2 \qquad b = \log 3$$

$$\log 6 = \log(2 \times 3)$$

$$= \log 2 + \log 3$$

$$= a + b$$

(ii) $\log 24$

$\log(2\times2\times2\times3)$

$\log 2^3 + \log 3$

$3\log 2 + \log 3$

$3a + b$

(iii) $\log 36$

$36 = 2\times18$

$2\times2\times9$

$2\times2\times3\times3$

$\log(2\times2\times3\times3)$

$\log(2^2\times3^2)$

$\log 2^2 + \log 3^2$

$2\log 2 + 2\log 3$

$2a + 2b$

$\log(2\times2\times3\times3) = \log 2 + \log 2 + \log 3 + \log 3$

Given $x = \log 5$ and $y = \log 3$ write in terms of x and y

(i) $\log 15$

$3 / 5$

Mult

Divide

15

$x = \log 5 \qquad y = \log 3$

$\log 15 = \log(3\times5)$

$= \log 3 + \log 5 = x + y$

(ii) $\log 45$

$3\ 15$

$\log(3\times3\times5)$

$\log 3 + \log 3 + \log 5$ 3/5/10

$2y + x$ $50 = 5 \times 10$)

(iii) $\log 50$

$\log (5 \times 10)$

$\log 5 + \log_{10} 10$

$x + 1$

Note $\log_{10} 10 = 1$ 3/5/10

$\log_{10} 2 = \log (10 \div 5)$

$= \log_{10} 10 - \log_{10} 5$

$= 1 - x$

Plot $y = 2^x$ for $0 \leq x \leq 3$

x	$y = 2^x$
0	$y = 2^0 = 1$
1	$y = 2^1 = 2$
2	$y = 2^2 = 4$
3	$y = 2^3 = 8$

x	$y = \log_2 x$
1	0
2	1
4	2
8	3

Plot $y = \log_2 x$