Simplify

$2^3 = 8$

power / indice / exponential

$2^3 = 8$ ← number

Base

$b^p = n$

$2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$4 \times 8 = 32 = 2^5$

$3^1 \times 3^2 = 3 \times 3 \times 3 = 3^3$

$a^m \times a^n = a^{m+n}$

$\frac{5^4}{5^2} = \frac{\cancel{5} \times \cancel{5} \times 5 \times 5}{\cancel{5} \times \cancel{5}} = 5^2$

$\frac{6^6}{6^3} = \frac{\cancel{6} \times \cancel{6} \times \cancel{6} \times 6 \times 6 \times 6}{\cancel{6} \times \cancel{6} \times \cancel{6}} = 6^3$

$\frac{a^m}{a^n} = a^{m-n}$

$\frac{2^3}{2^1} = \frac{8}{2}$ $\qquad$ $\frac{3^3}{3^1} = \frac{27}{3}$

$\frac{2^2}{2^1} = \frac{4}{2}$ $\qquad$ $\frac{3^2}{3^1} = \frac{9}{3}$

$\frac{2^1}{2^1} = \frac{2}{2}$ $\qquad$ $\frac{3^1}{3^1} = \frac{3}{3}$

$\frac{2^0}{2^1} = \frac{1}{2}$ $\qquad$ $3^0 = 1$

$$a^0 = 1$$

$$\frac{2^{-1}}{2^{1}} = \frac{\frac{1}{2}}{2}$$

$$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$$

Indices.

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$(2^3)^2 = 2^3 \times 2^3 = 2^6$$

$$(3^3)^3 = 3^3 \times 3^3 \times 3^3 = 3^9$$

$$(a^m)^n = a^{mn}$$

$5^2 = 25$ $\qquad$ $25^{\frac{1}{2}} = 5$

$3^2 = 9$ $\qquad$ $9^{\frac{1}{2}} = 3$

$4^3 = 64$ $\qquad$ $64^{\frac{1}{3}} = 4$

$2^4 = 16$ $\qquad$ $16^{\frac{1}{4}} = 2$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$(a^m)^n = a^{mn}$$

$$\frac{5^3}{5} = \frac{125}{5}$$

$$\frac{5^2}{5^1} = \frac{25}{5}$$

$$\frac{5^1}{5} = \frac{5}{5}$$

$$5^0 = 1$$

Simplify

(i) $(3x)^2 = 9x^2$

(ii) $\left(\frac{2}{5}\right)^3 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{2^3}{5^3}$

(iii) $(a+b)^2 = a^2 + 2ab + b^2$

Properties

(i) $(ab)^n = a^n b^n$

(ii) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(iii) $(a+b)^n \neq a^n + b^n$

Evaluations

(i) $3^2 = 9$

(ii) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ $\quad a^{-n} = \frac{1}{a^n}$

(iii) $36^{\frac{1}{2}} = \sqrt[2]{36} = 6$

$a^{-n} = \frac{1}{a^n}$ (iv) $8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

(v) $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$

$a^{-n} = \frac{1}{a^n}$ (vi) $49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}} = \frac{1}{7}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$ (vii) $64^{-\frac{1}{3}} = \frac{1}{64^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$

(viii) $\frac{1}{2^{-2}} = \frac{1}{\frac{1}{2^2}} = \frac{1}{1} \cdot \frac{2^2}{1} = 4$

$\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \times \frac{6}{5}$

$\frac{1}{1} \div \frac{1}{7} = \frac{1}{1} \times \frac{7}{1} = 7$

(ix) $\frac{1}{3^{-3}} = 3^3 = 27$ $a^{-n} = \frac{1}{a^n}$

$\frac{1}{a^{-n}} = a^n$

(x) $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$3^{-2} = \frac{1}{3^2}$

(xi) $\left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^2}$

$\frac{3^{-2}}{4^{-2}} = \frac{\frac{1}{3^2}}{\frac{1}{4^2}} = \frac{1}{\frac{9}{16}} = \frac{16}{9}$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

(xi) $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$ $\qquad \frac{2}{3} = \frac{1}{3} \times 2$

$= \left(\sqrt[3]{8}\right)^2$ $\qquad a^{mn} = (a^m)^n$

$= 4$

(xii) $4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = \left(\sqrt{4}\right)^3 = 8$

Simplify

(i) $x^{\frac{1}{2}} = \sqrt{x}$

(ii) $x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}} = \frac{1}{(\sqrt{x})^3}$

$\sqrt{x}\sqrt{x}\sqrt{x}$

$x\sqrt{x}$

$= \frac{1}{x\sqrt{x}}$

$\underline{\underline{x}}$ $\quad$ $\underline{\underline{in}}$ $\quad$ Po$\underline{\underline{w}}$er.

Solve for x

(i) $2^x = 8$

$2^x = 2^3$ $\qquad$ Same base = Same Base

$x = 3$

(ii) $3^x = \frac{9}{\sqrt{3}}$

$3^x = \frac{3^2}{3^{\frac{1}{2}}}$

$3^x = 3^{1\frac{1}{2}}$

$x = 1\frac{1}{2}$

(ii) $5^x = \frac{\sqrt{5}}{25}$

$$5^x = \frac{5^{\frac{1}{2}}}{5^2} \qquad \sqrt{x} = x^{\frac{1}{2}}$$

$$5^x = 5^{\frac{1}{2}-2} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$5^x = 5^{-1\frac{1}{2}}$$

$$x = -1\frac{1}{2}$$

(iii) $2^x = \frac{8}{\sqrt{2}}$

$$2^x = \frac{2^3}{2^{\frac{1}{2}}}$$

$$2^x = 2^{2\frac{1}{2}} \qquad x = 2\frac{1}{2}$$

(iv) $9^x = \frac{27}{\sqrt{3}}$

$$(a^m)^n = a^{mn} \qquad (3^2)^x = \frac{3^3}{3^{\frac{1}{2}}} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$3^{2x} = 3^{2\frac{1}{2}}$$

$$2x = 2\frac{1}{2}$$

$$x = 1\frac{1}{4}$$

(v) $$8^{x+1} = \frac{\sqrt{2}}{4}$$

$$\left(2^3\right)^{x+1} = \frac{2^{\frac{1}{2}}}{2^2}$$

$$2^{3x+3} = 2^{-1\frac{1}{2}}$$

$$3x+3 = -1\tfrac{1}{2}$$

$$3x = -4\tfrac{1}{2}$$

$$x = -1\tfrac{1}{2}$$

(vi) $$\frac{1}{9^{x-3}} = \frac{27}{\sqrt{3}}$$

$$\frac{1}{\left(3^2\right)^{x-3}} = \frac{3^3}{3^{\frac{1}{2}}}$$

$$\frac{1}{a^n} = a^{-n}$$

$$\frac{1}{3^{2x-6}} = 3^{2\frac{1}{2}}$$

$$3^{-2x+6} = 3^{2\frac{1}{2}}$$

$$-2x+6 = 2\tfrac{1}{2}$$

$$-2x = -3\tfrac{1}{2}$$

$$x = 1\tfrac{3}{4}$$

$$\frac{1}{9^{x-3}} = \frac{27}{\sqrt{3}}$$

$$9^{x-3} = \frac{\sqrt{3}}{27}$$

Quadratics.

Write in terms of 3^x

(i) 3^{x+1} $\quad a^{m+n} = a^m(a^n)$

$3^x(3)$

$= 3(3^x) = 3.3^x$

$$a^p a^q = a^{p+q}$$
$$\frac{a^p}{a^q} = a^{p-q}$$
$$(a^p)^q = a^{pq}$$
$$a^0 = 1$$
$$a^{-p} = \frac{1}{a^p}$$
$$a^{\frac{1}{q}} = \sqrt[q]{a}$$
$$a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$$
$$(ab)^p = a^p b^p$$
$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

(ii) $3^{x-2} = \dfrac{3^x}{3^2} = \dfrac{3^x}{9}$

(iii) $3^{x+3} = 3^x(3^3)$

$= 27(3^x)$

$(a^p)^q = a^{pq}$ $\quad$ (iv) $3^{2x} = (3^2)^x = 9^x$

$2x = 2 \times x$ $\qquad = (3^x)^2$

$= x \times 2 = x2$

Solve $2^{2x} - 10(2^x) + 16 = 0$

$10(2^x) \neq 20^x$ $\qquad 2(3^2) = 2(9) = 18$

$2^{2x} - 10(2^x) + 16 = 0$

$(2^x)^2 - 10(2^x) + 16 = 0$

$t = 2^x$

$t^2 - 10t + 16 = 0$

$$(t-2)(t-8)=0$$

$t=2$ $\quad$ $t=8$

$2^x=2$ $\quad$ $2^x=8$

$2^x=2^1$ $\quad$ $2^x=2^3$

$x=1$ $\quad$ $x=3$

$$3^{2x}-6(3^x)-27=0$$

$$(3^x)^2-6(3^x)-27=0$$

$$t^2-6t-27=0$$

$$(t+3)(t-9)=0$$

$t=-3$ $\quad$ $t=9$

$3^x=-3$ $\quad$ $3^x=3^2$

$3^{-1}=\frac{1}{3^1}$ $\quad$ Impossible $\quad$ $x=2$

$(\text{Positive})^n = \text{positive}$

Solve $\quad$ $2^{2x+1}-9(2^x)+4=0$

$2^{2x+1}=2(2^{2x})$ $\quad$ a^{m+n}

$=2(2^x)^2$

$$2(2^x)^2-9(2^x)+4=0$$

$$2t^2-9t+4=0$$

$$2t^2 - 8t - 1t + 4 = 0$$

$$2t(t-4) - 1(t-4) = 0$$

$$(t-4)(2t-1) = 0$$

$$t = 4 \qquad t = \frac{1}{2}$$

$$2^x = 4 \qquad 2^x = \frac{1}{2}$$

$$2^x = 2^2 \qquad 2^x = 2^{-1}$$

$$x = 2 \qquad x = -1$$

Solve $\quad 3^{2x+1} - 8(3^x) - 3 = 0$

$$3^{2x+1} = 3^1(3^{2x}) \qquad a^{m+n} = a^m . a^n$$

$$= 3(3^x)^2$$

$$3(3^x)^2 - 8(3^x) - 3 = 0$$

$$t = 3^x$$

$$3t^2 - 8t - 3 = 0$$

$$(3t+1)(t-3) = 0$$

$$t = -\frac{1}{3} \qquad t = 3$$

$$3^x = -\frac{1}{3} \qquad 3^x = 3^1$$

Impossible $\qquad x = 1$

$f(x) = 2^x$ simplify

$f(x+1) = 2^{x+1} = 2(2^x)$

$g(x) = 3^x$ simplify

$$a^m . a^n = a^{m+n}$$

$$g(x+2) = 3^{x+2}$$
$$= 3^x(3^2)$$
$$= 9(3^x)$$

$$\frac{a^m}{a^n} = a^{m-n}$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^{mn} = (a^m)^n$$

$P_x = 5^x$ simplify

P_{x-1}

$$P_x = 5^x$$
$$P_{x-1} = 5^{x-1} = \frac{5^x}{5}$$

$$a^{m-n} = \frac{a^m}{a^n}$$

$U_n = 2^n + 3^n$ simplify

(i) U_{n+1}

(ii) U_{n+2}

$$U_n = 2^n + 3^n$$

(i) $U_{n+1} = 2^{n+1} + 3^{n+1}$ $\quad a^{m+n} = a^m \cdot a^n$

$$= 2^n(2) + 3^n(3)$$
$$= 2(2^n) + 3(3^n)$$

(ii) $U_{n+2} = 2^{n+2} + 3^{n+2}$

$$= 2^2(2^n) + 3^2(3^n)$$
$$= 4(2^n) + 9(3^n)$$

$U_n = 3^n + 5^n$ find

(i) U_{n+1}

(ii) U_{n+2}

Prove $U_{n+2} - 8U_{n+1} + 15U_n = 0$

$U_n = 3^n + 5^n$

$U_{n+1} = 3^{n+1} + 5^{n+1}$ $\qquad a^m \cdot a^n = a^{m+n}$

$= 3^n(3) + 5^n(5)$

$= 3(3^n) + 5(5^n)$

$U_{n+2} = 3^{n+2} + 5^{n+2}$

$= 3^2(3^n) + 5^2(5^n)$

$= 9(3^n) + 25(5^n)$

$U_{n+2} - 8U_{n+1} + 15U_n = 0$

Let $a = 3^n$ and $b = 5^n$

$9a + 25b - 8(3a + 5b) + 15(a+b)$

$9a + 25b - 24a - 40b + 15a + 15b$

$= 0$

$U_n = 3^n + 7^n$ prove $U_{n+2} - 10U_{n+1} + 21U_n = 0$

$U_{n+1} = 3^{n+1} + 7^{n+1}$

$= 3(3^n) + 7(7^n)$

$U_{n+2} = 3^{n+2} + 7^{n+2}$

$= 9(3^n) + 49(7^n)$ $\qquad a = 3^n$

$b = 7^n$

$9a + 49b - 10(3a + 7b) + 21(a+b)$

$9a + 49b - 30a - 70b + 21a + 21b = 0$

Plot $f(x) = 2^x$ in the domain $-2 \le x \le 2$.

$f(-2) = 2^{-2} = \frac{1}{4}$

$f(-1) = 2^{-1} = \frac{1}{2}$

$f(0) = 2^0 = 1$

$f(1) = 2$

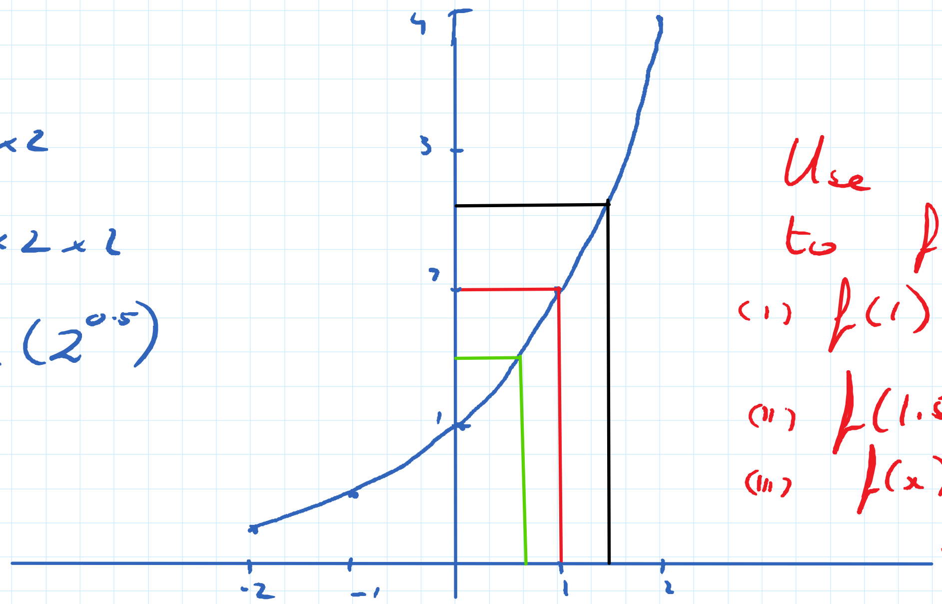
$f(2) = 2^2 = 4$

$2^2 = 2 \times 2$

$2^3 = 2 \times 2 \times 2$

$2^{1.5} = 2(2^{0.5})$

$2^{1.5555}$



Use graph to find

(i) $f(1) = 2$

(ii) $f(1.5) = 2.6$

(iii) $f(x) = 1.5$ find x

$x = 0.7$

$2^x = 1.5$

$2^x = 2$ $\quad$ $2^x = 4$ $\quad$ $2^x = 8$

$x = 1$ $\quad$ $x = 2$ $\quad$ $x = 3$