Max / Min Problems

$A = xy$ and $x + y = 12$. Find the maximum value for A.

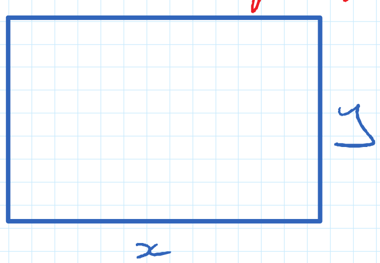
$$A = xy$$
$$x + y = 12$$
$$y = 12 - x$$
$$A = xy$$
$$A = x(12 - x)$$
$$A = 12x - x^2$$
$$\frac{dA}{dx} = 12 - 2x = 0$$
$$x = 6$$
$$y = 12 - 6 = 6$$
$$A = 36$$

$A = xy$ $\qquad$ $x + y = 12$

$A = 6(6)$ $\qquad$ $x = 7 \quad y = 5 \quad = 35$

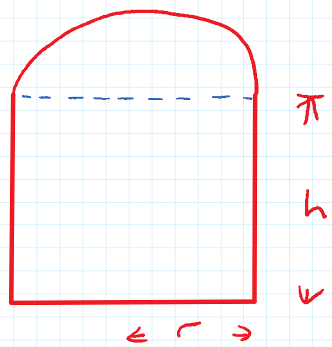
$= 36$ $\qquad$ $x = 10 \quad y = 2 \quad = 20$

A farmer has 20m of wire to form a rectangular field. What are dimensions of field with maximum area?



$$A = xy$$
$$2x + 2y = 20$$
$$x + y = 10$$
$$y = 10 - x$$
$$A = x(10 - x)$$
$$A = 10x - x^2$$
$$\frac{dA}{dx} = 10 - 2x = 0$$
$$x = 5m \qquad y = 5m$$

$$A = 10(5) - 5^2 = 25m^2$$



The perimeter is 8m. Find maximum area.

$$A = 2rh + \frac{1}{2}\pi r^2$$

$$Arc = \pi r$$

$$2h + 2r + \pi r = 8$$

$$2h = 8 - 2r - \pi r$$

$$A = r(8 - 2r - \pi r) + \frac{1}{2}\pi r^2$$

$$A = 8r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$$

$$A = 8r - 2r^2 - \frac{1}{2}\pi r^2$$

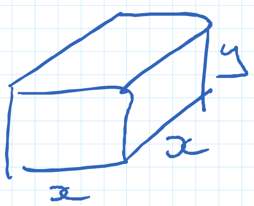
$$\frac{dA}{dr} = 8 - 4r - \pi r = 0$$

$$8 = r(4 + \pi)$$

$$\frac{8}{4+\pi} = r$$

Box

A closed box with a square base has a volume of $288m^3$. Find dimensions of box with minimum area.



$$A = 2(lb + lh + bh)$$

$$A = 2(x^2 + xy + xy)$$

$$A = 2x^2 + 4xy$$

$$V = 288 \qquad V = l \times b \times h$$

$$x^2 y = 288$$

$$y = \frac{288}{x^2}$$

$$V = 2x^2 + 4x\left(\frac{288}{x^2}\right)$$

$$V = 2x^2 + 1152x^{-1}$$

$$\frac{dV}{dx} = 4x - 1152x^{-2} = 0$$

$$4x = \frac{1152}{x^2}$$

$$4x^3 = 1152$$

$$x^3 = 288$$

$$x = \sqrt[3]{288}$$

$$x = 6.6$$

A solid cylender has a volume of 288π cm^3. Find dimensions of area is a minimum.

$$V = 288\pi$$

$$\cancel{\pi} r^2 h = 288\cancel{\pi}$$

$$h = \frac{288}{r^2}$$

$$A = 2\pi r h + 2\pi r^2$$

$$A = 2\pi r\left(\frac{288}{r^2}\right) + 2\pi r^2$$

$$A = 576\pi r^{-1} + 2\pi r^2$$

$$\frac{dA}{dr} = -576\cancel{\pi} r^{-2} + 4\cancel{\pi} r = 0$$

$$4r = \frac{576}{r^2}$$

$$4r^3 = 576$$

$$r^3 = 144$$

$$r = 5.2 \text{ cm}$$

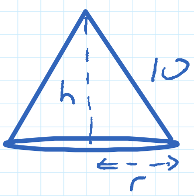
$$h = \frac{288}{(5.2)^2} = 10.5 \text{ cm}$$

A closed cylinder has surface area of 24π m^2. Find maximum volume.

$$2\cancel{\pi}rh + 2\cancel{\pi}r^2 = 24\cancel{\pi}$$
$$rh + r^2 = 12$$
$$rh = 12 - r^2$$
$$h = \frac{12 - r^2}{r}$$

$$V = \pi r^2 h$$
$$V = \pi r^{\cancel{2}}\left(\frac{12 - r^2}{\cancel{r}}\right)$$
$$V = \pi\left(12r - r^3\right)$$
$$\frac{dV}{dr} = \pi\left(12 - 3r^2\right) = 0$$
$$12 = 3r^2$$
$$r^2 = 4$$
$$r = 2$$
$$V = \pi\left(12(2) - 2^3\right) = 16\pi\ m^3$$

A cone has a slant height of 10 m. Find maximum volume.



$$h^2 + r^2 = 10^2$$
$$r^2 = 100 - h^2$$

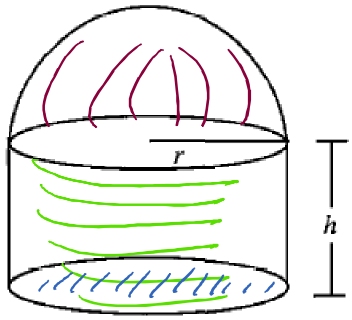
$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi\left(100 - h^2\right)h$$
$$V = \frac{1}{3}\pi\left(100h - h^3\right)$$
$$\frac{dV}{dh} = \frac{1}{3}\pi\left(100 - 3h^2\right) = 0$$

$$3h^2 = 100$$
$$h^2 = \frac{100}{3}$$
$$h = \frac{10}{\sqrt{3}}$$

A tank with a base is made from thin uniform metal. The tank standing on level ground is in the shape of an upright circular cylinder and hemispherical top with radius of length r metres. The height of the cylinder is h metres.

(i) If the total surface area of the tank is 45π m^2, express h in terms of r.

(ii) Find the value of h and of r for which the tank has maximum volume.



$$A = 45\pi$$
$$A = \pi r^2 + 2\pi r h + 2\pi r^2$$
$$3\cancel{\pi} r^2 + 2\cancel{\pi} r h = 45\cancel{\pi}$$
$$2rh = 45 - 3r^2$$
$$h = \frac{45-3r^2}{2r}$$

$$V = \pi r^2 h + \frac{2}{3}\pi r^3$$
$$V = \pi r^{\cancel{2}}\left(\frac{45-3r^2}{2\cancel{r}}\right) + \frac{2}{3}\pi r^3$$
$$V = \frac{\pi}{2}\left(45r - 3r^3\right) + \frac{2}{3}\pi r^3$$

$$\frac{dV}{dr} = \frac{\pi}{2}\left(45 - 9r^2\right) + 2\pi r^2 = 0$$
$$\pi\left(45 - 9r^2\right) + 4\pi r^2 = 0$$
$$45 - 9r^2 + 4r^2 = 0$$
$$45 = 5r^2$$
$$r^2 = 9$$
$$r = 3 \text{ cm}$$

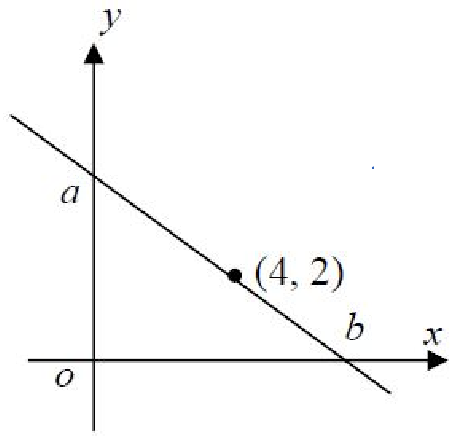
$$h = \frac{45-3r^2}{2r}$$

$$h = \frac{45 - 3(3)^2}{6} = 3\text{cm}$$

A line passes through the point (4, 2) and has slope m, where $m < 0$. The line intersects the axes at the points a and b.

(i) Find the co-ordinates of a and b, in terms of m.

(ii) Hence, find the value of m for which the area of triangle aob is a minimum.



$$y - 2 = m(x - 4)$$

a - y-axis $\quad x = 0$

$$y - 2 = -4m$$

$$y = 2 - 4m$$

b - x-axis $\quad y = 0$

$$-2 = mx - 4m$$

$$4m - 2 = mx$$

$$x = \frac{4m-2}{m}$$

$$A = \frac{1}{2}\left(\frac{4m-2}{m}\right)(2-4m)$$

$$A = \frac{1}{2}\left(\frac{8m - 16m^2 - 4 + 8m}{m}\right)$$

$$A = \frac{1}{2}\left(\frac{16m - 16m^2 - 4}{m}\right)$$

$$A = \frac{8m - 8m^2 - 2}{m}$$

$$A = 8 - 8m - \frac{2}{m}$$

$$A = 8 - 8m - 2m^{-1}$$

$$\frac{dA}{dm} = -8 + 2m^{-2} = 0$$

$$\frac{2}{m^2} = 8$$

$$2 = 8m^2$$

$$m^2 = \frac{1}{4}$$

$$m = \pm \frac{1}{2}$$

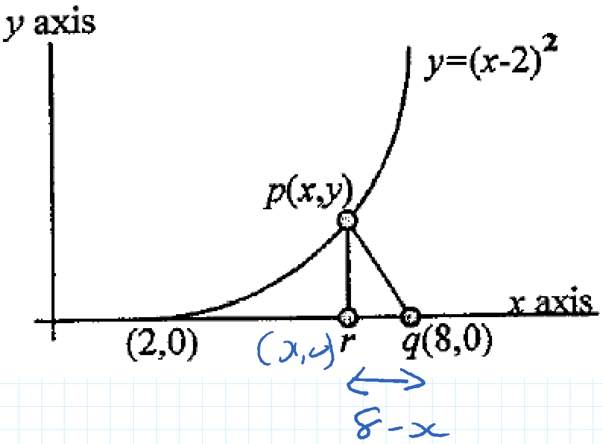
$$m = -\frac{1}{2}$$

$p(x, y)$ is a point on the curve $y = (x-2)^2$ in the domain $2 < x < 8$.
q is the point $(8, 0)$ and $pr \perp rq$.

Express, in terms of x, the area of the triangle prq.

What value of x maximises the area of triangle prq?

Find the maximum area of triangle prq.



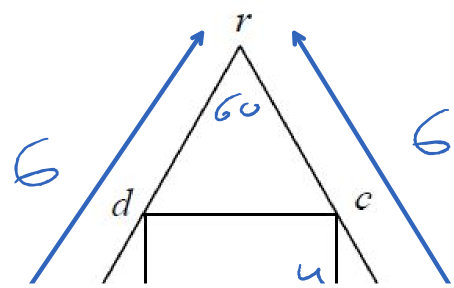
$$A = \frac{1}{2}(8-x)y$$

$$A = \frac{1}{2}(8-x)(x-2)^2$$

$$A = \frac{1}{2}(8-x)(x^2-4x+4)$$

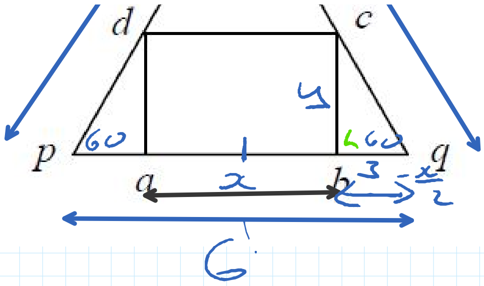
pqr is an equilateral triangle of side 6 cm.
$abcd$ is a rectangle inscribed in the triangle as shown.
$|ab| = x$ cm and $|bc| = y$ cm.

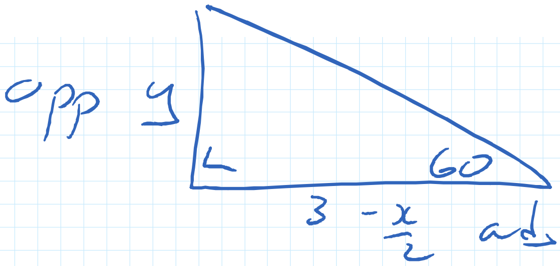
(i) Express y in terms of x.



(i) Express y in terms of x.

(ii) Find the maximum possible area of $abcd$.





$$\tan 60 = \frac{y}{3 - \frac{x}{2}}$$

$$\sqrt{3}\left(3 - \frac{x}{2}\right) = y$$

$$A = xy$$

$$= x\left(\sqrt{3}\left(\frac{6-x}{2}\right)\right)$$

$$A = \frac{\sqrt{3}}{2}\left(6x - x^2\right)$$

$$\frac{dA}{dx} = \frac{\sqrt{3}}{2}\left(6 - 2x\right) = 0$$

$$x = 3$$

$$A = \frac{\sqrt{3}}{2}\left(18 - 9\right) = \frac{9\sqrt{3}}{2}\ \text{cm}^2.$$

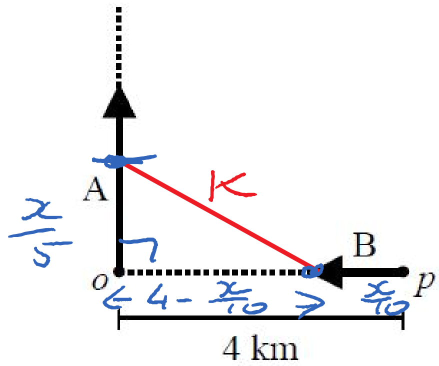
The point p is 4 km due east of the point o.

At noon, A leaves o and travels north at a steady speed of 12 km/h. At the same time, B leaves p and travels towards o at a steady speed of 6 km/h.

(i) Write down expressions in x for the distances that A and B will each have travelled at x minutes after noon.

(ii) Find an expression in x for the distance that B will be from A at x minutes after noon.

(iii) At how many minutes after noon will B be closest to A?



Speed Time Dist

(iii) At how many minutes after noon will B be closest to A?

	Speed	Time	Dist
A	$\frac{1}{5}$ km/m	x	$\frac{x}{5}$
B	$\frac{1}{10}$ km/m	x	$\frac{x}{10}$

$$K^2 = \left(\frac{x}{5}\right)^2 + \left(\frac{40-x}{10}\right)^2$$

$$K^2 = \frac{x^2}{25} + \frac{1600 - 80x + x^2}{100}$$

$$K^2 = \frac{4x^2 + 1600 - 80x + x^2}{100}$$

$$K = \left(\frac{5x^2 - 80x + 1600}{100}\right)^{\frac{1}{2}}$$

$$\frac{dK}{dx} = \frac{10x - 80}{100}\left(\frac{1}{2}\left(\frac{5x^2 - 80x + 1600}{100}\right)^{-\frac{1}{2}}\right) = 0$$

$$= \frac{10x - 80}{100}\left(\frac{1}{2}\right)\frac{1}{\left(\frac{5x^2 - 80x + 1600}{100}\right)^{\frac{1}{2}}} = 0$$

$$10x - 80 = 0$$

$$x = 8$$