Prob and Stats

EKKER

Permutations = order is vital.

Write out all possible permutations of 1, 2, 3, each digit used only once.

1 2 3	2 1 3	3 1 2
1 3 2	2 3 1	3 2 1

Writing out all answers is called 'sample space'.

Fundamental Principal of Counting.

I can go for tea in 2 ways.

I go to a game in 3 ways.

How many ways can I go for

(i) tea and a game?

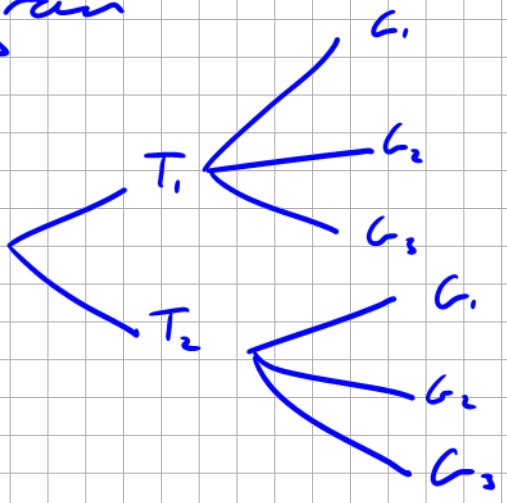
Tea 2 Game 3

AND = MULTIPLY

T_1 T_2 = Tea $G_1 G_2 G_3$ = Games

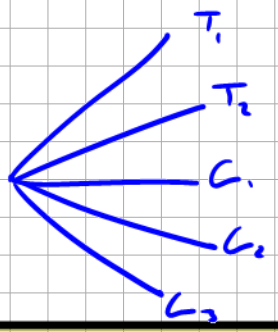
$T_1 G_1$	$T_2 G_1$
$T_1 G_2$	$T_2 G_2$
$T_1 G_3$	$T_2 G_3$

Tree diagram



$2 \times 3 = 6$

(11) tea or a game?



OR = ADD

FPC

Given one task can be done in m ways and second in n ways then both tasks can be done in $m \times n$ ways.

How many 4 digit numbers can be formed from 2.3.4.5 each digit used only once?

$4 \times 3 \times 2 \times 1 = 24$ ~~1 2 3 4 5~~

How many ways can 6 horses finish a six horse race?

$6 \times 5 \times 4 \times 3 \times 2 \times 1$ ~~A B C D E F~~

$= 720$

How many ways can 10 horses finish a race?

$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$3,628,800$

$n!$

$n \in N$ next down $n-1$

$n!$ = n factorial

$n! = n(n-1)(n-2) \ldots\ldots 3(2)(1)$

$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$100! = 100 \times 99 \times \text{-----} \times 3 \times 2 \times 1$

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Simplify

$$\frac{n!}{(n-2)!} \qquad n \in N$$

Factorials = break bigger one down

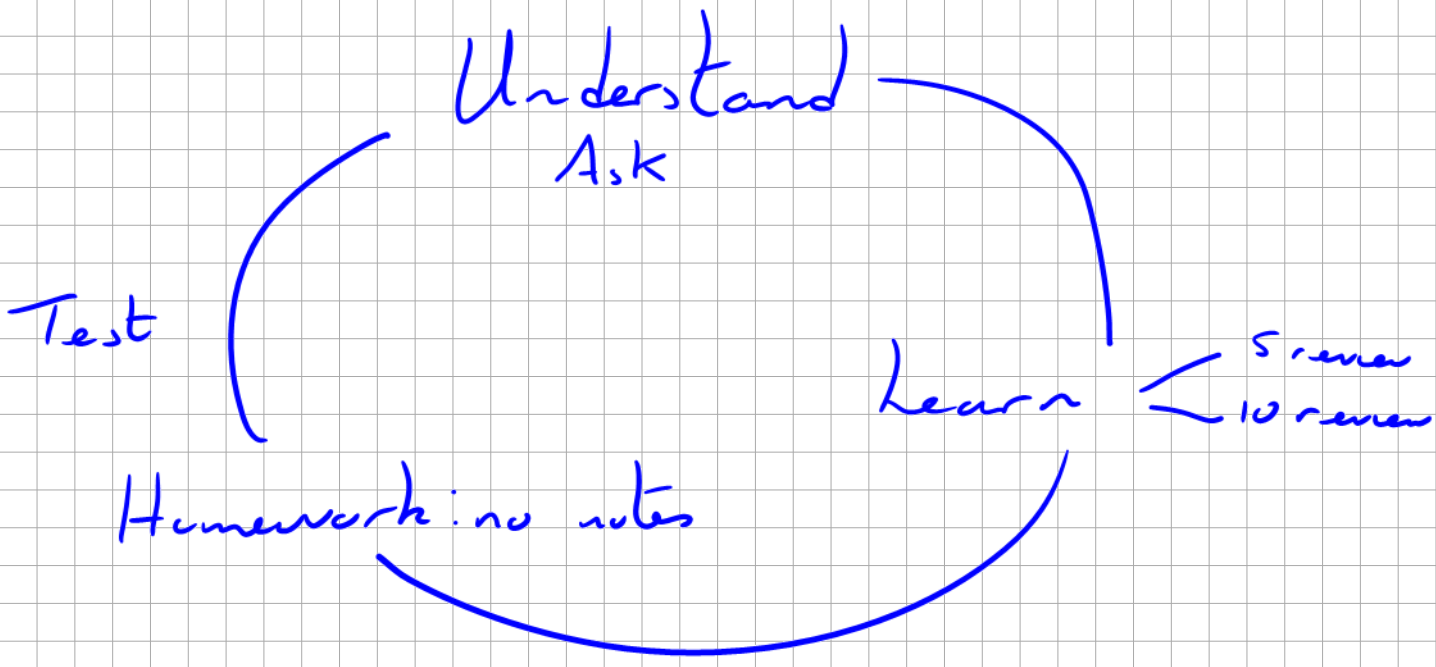
$n+2, n+1, n, n-1, n-2, \ldots$

$$\frac{n!}{(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$$

$$= n^2 - n$$

$$\frac{5!}{3!} = \frac{5 \times 4 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1}}$$

$$= \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}}$$



Solve for n

$$\frac{(n+1)!}{(n-1)!} = 20.$$

$$\frac{(n+1)\,n\,(n-1)!}{(n-1)!} = 20$$

$$n^2 + n = 20$$

$$n^2 + n - 20 = 0$$

$$(n+5)(n-4) = 0$$

$n = -5$ $\quad$ $n = 4$

Find factors !

$$(2n+1)! + (2n-1)!$$

$$(2n+1)\,2n\,(2n-1)! + (2n-1)!$$

$$(2n-1)!\,[(2n+1)\,2n + 1]$$

$$(2n-1)!\,[4n^2 + 2n + 1].$$

$ax + ay$
$a(x+y)$

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AND	=	MULT
OR	=	ADD

How many 3 letter words can be made from A, B, C, D, E. each letter used only once?

$\underline{5} \times \underline{4} \times \underline{3} = 60$ ~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~

How many of these

(i) start with a vowel?

$\underset{AE}{\underline{2}} \times \underline{4} \times \underline{3} = 24$ ~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~

Alt $\frac{2}{5}$ (60) $\Rightarrow$ 2 out 5 are vowels

(ii) start and end in vowel?

$\underset{AE}{\underline{2}} \times \underline{4} \times \underline{1} = 8$ ~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~

$\underset{AE}{\underline{2}} \times \underline{3} \times \underset{E}{\underline{1}} = 6$ ~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~

How many 4 digit numbers can be formed from 1, 2, 3, 4, 5, 6 each digit used only once?

$\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} = 360$ 1 2 3 ~~4~~ ~~5~~ ~~6~~

How many of these are

(i) over 5,000 ?

$\underset{56}{2} \times \underline{\quad} \times \underline{\quad} \times \underline{\quad} = 120$ 1 2 3 4 ~~5~~ ~~6~~

(ii) odd ?

$\underline{\quad} \times \underline{\quad} \times \underline{\quad} \times \underset{135}{3} = 180$ ~~1~~ 2 3 4 5 6

(iii) over 5,000 and odd ?

$\underset{56}{2} \times \underline{\quad} \times \underline{\quad} \times \underset{135}{3}$ 1 2 3 4 5 6

$\underset{56}{2} \times \underline{\quad} \times \underline{\quad} \times \underset{13}{2}$ 1 2 3 4 ~~5~~ ~~6~~

$\underset{5}{1} \times \underline{\quad} \times \underline{\quad} \times \underset{13}{2} = 24$ ~~1~~ 2 3 4 ~~5~~ ~~6~~

+

or $\underset{6}{1} \times \underline{\quad} \times \underline{\quad} \times \underset{135}{3} = \frac{36}{60}$ 1 2 3 4 ~~5~~ ~~6~~

How many 3 digit numbers can be formed from 0, 1, 2, 3, 4 each digit used only once ?

$\underline{5} \times \underline{4} \times \underline{3} = 60$ ~~0~~ 1 2 3 4

$\underset{1234}{4} \times \underline{4} \times \underline{3} = 48$ 0 1 2 3 ~~4~~

How many 3 digit codes can be formed from 0, 1, 2, 3, 4?

$5 \times 5 \times 5 = 125$ 0 1 2 3 4

Together

How many ways can 6 girls stand in Line?

$6! = 720$

2 are sisters. How many ways can they stand in Line if sisters are (i) always together?

$5! = 120 \times 2 = 240$

(ii) never together?

1 — — — — 1 ??

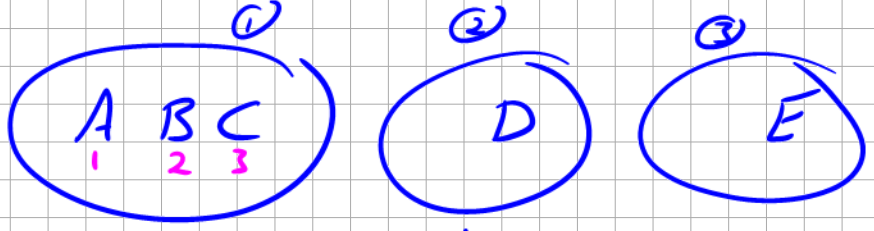
Not together = total - together

$= 720 - 240$

$= 480$

Together = bubbles

How many 5 letter words can be formed from A, B, C, D, E where A B C are not all together each letter used only once?



Total - together

$5! - 3!\,3!$

Identical Objects

How many 3 letter words can be formed from a, a, b each letter used only once?

$3 \times 2 \times 1 = 6$

a a b a b a b a a

~~a a b~~ ~~a b a~~ ~~b a a~~

4 letters using a, a, a, b only once?

a a a b a a b a a b a a

b a a a

$$\frac{4!}{3!}$$

Order of identical.

aaab
aaab
aaab
aaab
aaab
aaab

3! changes

CAVAN - 5 letter words, each letter used once.

$$\frac{5!}{2!} = 60$$

NAVAN.

$$\frac{5!}{2!.2!} = 30$$

$2!$ → As, $2!$ → Ns

AAABB

$$\frac{5!}{3!\,2!} = 10$$

How many 4 letter words can be formed from a, b, c, d so that each letter is used only once and a is always before b.

$$\underline{1} \times \underline{1}_{B} \times \underline{2} \times \underline{1} = 2 \quad AbcD$$

+

or

$$\underline{3} \times \underline{2} \times \underline{1} \times \underline{1}_{B} = 6 \quad AbcD$$

+

or

$$\underline{2} \times \underline{1} \times \underline{1}_{B} \times \underline{2}_{CD} = 4 \quad AbCD$$

12.

$$\underline{1}_{A} \times \underline{3} \times \underline{2} \times \underline{1}$$

$$\underline{2}_{CD} \times \underline{1}_{A} \times \underline{2} \times \underline{1}$$

$$\underline{2} \times \underline{1} \quad \underline{1}_{A} \quad \underline{1}_{B}$$

How many numbers over 300 can be formed from 1, 2, 3, 4 each digit used only once?

$$\underline{2} \times \underline{3} \times \underline{2} = 12 \quad 12\cancel{4}$$

+

$$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$$

36

${}^nP_r$ - do not worry.

How many 2 letter words can be formed from A, B, C, D each letter used only once?

$4 \times 3 = 12$ ABCD

$n = 4 \qquad r = 2$

${}^nP_r = {}^4P_2 = 12.$

Defn ${}^nP_r = \dfrac{n!}{(n-r)!}$

Given ${}^nP_2 = 20$ find n.

${}^nP_2 = \dfrac{n!}{(n-2)!} = 20$

$= \dfrac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$

$= n^2 - n = 20$

$n^2 - n - 20 = 0$

$(n-5)(n+4) = 0$

$n = 5 \qquad \cancel{n = -4}$