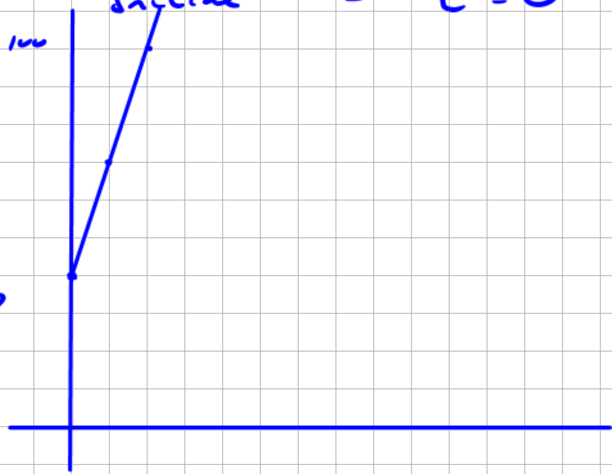
Patterns

A plumber charges €40 call out fee and €30 per hour. Show on a table and a graph. Form a function for fees.

Time = independent event
Money = dependent
Initial = $t = 0$

Time	Money	
0	40	
		) 30
1	70	
		) 30
2	100	
		) 30
3	130	
x	$f(x) = 30x + 40$	



Linear = gap is same

× ○ □ × ○ □ ×

What shape is in 51^{st} place?
Where is 61^{st} square?

× ○ □ × ○ □
T_1 T_2 T_3 T_4 T_5 T_6

Square $T_n = 3n$ $\quad 51^{st}$ = square
Circle $T_n = 3n - 1$
× $T_n = 3n - 2$

$T_{61} = 3(61) = 183$

Sequence

Sequence is an array of terms.

$T_1, T_2, T_3 \ldots T_n, T_{n+1}, T_{n+2}$

T_1 = first term

$\boxed{T_n = \text{general term}}$ $\boxed{n \in N}$

$T_n = U_n$.

$T_n = 5n + 3$ find first 3 terms.

$T_n = 5n + 3$ is same idea as

$f(x) = 5x + 3$

$T_n = 5n + 3$

$$\left.\begin{aligned} T_1 &= 5(1) + 3 = 8 \\ T_2 &= 5(2) + 3 = 13 \\ T_3 &= 5(3) + 3 = 18 \end{aligned}\right\} \begin{matrix} 5 \\ 5 \end{matrix}$$

$T_n = 2n - 1$ find

(i) T_{50}

(ii) n when $T_n = 1031$ (which term has value of 1031?)

$T_n = 2n - 1$

(i) $T_{50} = 2(50) - 1 = 99$

(ii) $T_n = 1031$

$2n - 1 = 1031$

$2n = 1032$

$n = 516$

T_n = general term

$\boxed{T_n = \text{RULE for sequence}}$

EKKER

Given $T_n \Rightarrow$ find sequence

Given sequence $\Rightarrow$ find T_n.

Recurring Relations

$U_1 = 1$ and $U_2 = 1$ given

$U_{n+1} = U_n + U_{n-1}$ find U_5.

$$U_1, U_2, \ldots U_{n-1}, U_n, U_{n+1}$$

$$U_{n+1} = U_n + U_{n-1}$$

$$U_3 = U_2 + U_1 = 1+1 = 2$$

$$U_4 = U_3 + U_2 = 1+2 = 3$$

$$U_5 = U_4 + U_3 = 2+3 = 5$$

$$1,\ 1,\ 2,\ 3,\ 5,\ 8$$

This is called the Fibonacci Sequence.

$U_{n+2} = 2U_{n+1} + U_n$ given $U_1 = 2$ and $U_2 = 4$ find U_4.

$$U_3 = 2U_2 + U_1$$

$$= 2(4) + 2 = 10$$

$$U_4 = 2U_3 + U_2$$

$$2(10) + 4 = 24$$

Note $U_{n+1} \neq U_n + 1$

U_{n+1} = next term up from U_n in the sequence.

EKKER

$U_n = 2^n + 9^n$ prove

$U_{n+2} - 11U_{n+1} + 18U_n = 0.$

$U_n = 2^n + 9^n$

a^{p+q}

$a^p \cdot a^q$

$U_{n+1} = 2^{n+1} + 9^{n+1}$

$= 2(2^n) + 9(9^n)$

$U_{n+2} = 2^{n+2} + 9^{n+2}$

$= 4(2^n) + 81(9^n)$

Let $a = 2^n$ $b = 9^n$

$4a + 81b - 11(2a + 9b) + 18(a+b)$

$\cancel{4a} + 81b - \cancel{22a} - 99b + \cancel{18a} + 18b = 0$

Increasing – Decreasing.

Increasing $\Rightarrow T_{n+1} > T_n$

$\Rightarrow$ $\boxed{\frac{T_{n+1}}{T_n} > 1 \Rightarrow \text{Increasing}}$

$\boxed{\frac{T_{n+1}}{T_n} < 1 \Rightarrow \text{Decreasing}}$

$T_n = \frac{2n+1}{n-1}$ is the sequence increasing.

$T_n = \frac{2n+1}{n-1}$

$$T_{n+1} = \frac{2(n+1)+1}{n+1-1} = \frac{2n+3}{n}$$

$$\frac{T_{n+1}}{T_n} = \frac{2n+3}{n} \cdot \frac{n-1}{2n+1}$$

$$= \frac{2n^2+n-3}{2n^2+n} < 1$$

$\Rightarrow$ decreasing

Since top is smaller than bottom (-3). $n \in \mathbb{N}$.

$T_n = \frac{3n-1}{2n+5}$ is it increasing?

$$T_{n+1} = \frac{3(n+1)-1}{2(n+1)+5} = \frac{3n+2}{2n+7}$$

$$\frac{T_{n+1}}{T_n} = \frac{3n+2}{2n+7} \cdot \frac{2n+5}{3n-1}$$

$$= \frac{6n^2+19n+10}{6n^2+19n-7} > 1 \Rightarrow \text{increasing}$$

It's >1 since $10 > -7$

<u>Quadratic</u>

Find T_n of $3, 7, 13, 21, 31$

$$3, \quad 7, \quad 13, \quad 21, \quad 31$$

$$4 \quad 6 \quad 8 \quad 10$$

$$2 \quad 2 \quad 2$$

Gap of gap is same

Gap is same = linear = arithmetic

Gap of gap same = quadratic

EKKER

Let $T_n = an^2 + bn + c$

Gap of gap is $2 \Rightarrow a = 1$

" " " is $4 \Rightarrow a = 2.$

" " " " $2x \Rightarrow a = x$

$$\underset{T_1}{3}, \underset{T_2}{7}, \underset{T_3}{13}, \underset{T_4}{21}$$

$T_n = n^2 + bn + c$

$T_1 = 1 + b + c = 3$

$b + c = 2$

$T_2 = 4 + 2b + c = 7$

$2b + c = 3$

$$\begin{array}{r} b + c = 2 \\ 2b + c = 3 \\ \hline b = 1 \\ c = 1 \end{array}$$

2, 5, 10, 17, 26 find T_n.

$$\underbrace{2,\ 5}_{3},\ \underbrace{5,\ 10}_{5},\ \underbrace{10,\ 17}_{7},\ 26, \ldots$$

$$\underbrace{3 \quad 5}_{2} \quad \underbrace{5 \quad 7}_{2}$$

Quad $T_n = n^2 + an + b$

$T_1 = 1 + a + b = 2$

$a + b = 1$

$T_2 = 4 + 2a + b = 5$

$2a + b = 1$

$$\begin{array}{r} a + b = 1 \\ \hline a = 0 \\ b = 1 \end{array}$$

$T_n = n^2 + 1$

$T_5 = 5^2 + 1 = 26$

$= 7n - \cancel{n^2} + \cancel{n^2} - 9n + 8$

$= 8 - 2n$