A plumber charges €80 call out and €50 per hour. Show on a table and form a function for fees.

	Time	Money	
Initial	0	80	> 50
	1	130	> 50
	2	180	
	x	$y = f(x) = 50x + 80$	

Money = dependent

Time = independent

$y = mx + c$

Money is a function of time.

Input → Processes → Output

Domain → Function → Range

x → $f(x) = \text{rule}$ → y

$f(x) = 3x + 1$ find

(i) $f(5)$

(ii) x when $f(x) = 17$.

(i) $f(5) = 3(5) + 1 = 16$

(ii) $3x + 1 = 17$

$3x = 16$

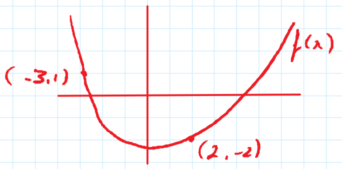
$x = \frac{16}{3}$

$g(x) = ax + b$ find a and b when $g(0) = 7$ and $g(1) = 9$.

$g(0) = b = 7$

$g(1) = a + 7 = 9$

$a = 2$



$f(x) = x^2 + ax + b$

find a and b.

$y = f(x)$

$y = x^2 + ax + b$

$(-3, 1)$ — x, y: $1 = 9 - 3a + b$

$3a - b = 8$

$(2, -2)$ — x, y: $-2 = 4 + 2a + b$

$2a + b = -6$

$3a - b = 8$

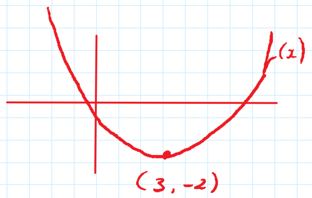
$5a = 2$

$a = \frac{2}{5}$

$2\left(\frac{2}{5}\right) + b = 6$

$4 + 5b = 30$

$b = \frac{26}{5}$



$f(x) = x^2 + ax + b$

Find a and b.

$y = (x-3)^2 - 2.$

$y = x^2 - 6x + 9 - 2$

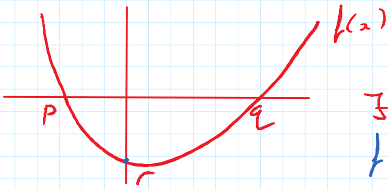
$y = x^2 - 6x + 7$

$y = x^2 - 6x + 7$

$y = x^2 - 6x + 9 + 7 - 9$

$y = (x-3)^2 - 2$

Minimum $(3, -2)$



$f(x) = x^2 - 2x - 3$

Find p, q and r.

$f(x) = y$

$y = x^2 - 2x - 3$

p and q are on x-axis

x-axis $= y = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \qquad x = -1$

$(3, 0) \qquad (-1, 0)$

r on y-axis $\quad x = 0$

$y = -3 \qquad (0, -3)$

$g(x) = 2x - 1$ in the domain $-1 \le x \le 2$ find the range.

$g(x) = 2x - 1 \qquad m = 2$

$g(-1) = 2(-1) - 1 = -3$

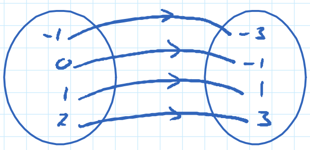
$g(0) = -1$

$g(1) = 1$

$g(2) = 3$

Forms couples and show in Arrow diagram. (x, y)

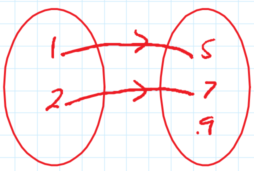
$(-1, -3) \quad (0, -1) \quad (1, 1) \quad (2, 3)$



Bijective function.

Couples (x, y) = (input, output)

Function every input is used and is used only once.



Is it a function?

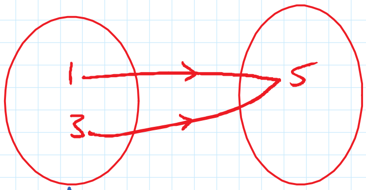
Which type?

State (i) Domain

(ii) Range

(iii) Co-domain.

Yes : 1st Used and used once.
Injective ⇒ 2nd set 9 is idle.
Domain = { 1, 2 } = 1st set
Range = { 5, 7 } = 2nd set used
Co-domain = { 5, 7, 9 } = 2nd set.



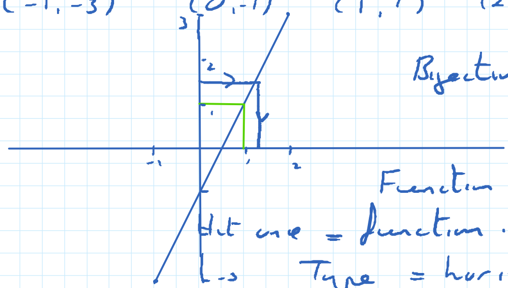
What type of function.

Surjective ⇒ many to one

Note : Bijective is said to be injective and surjective.

Draw $f(x) = 2x-1$ in domain $-1 \le x \le 2$. Which type of function.

(−1, −3) (0, −1) (1, 1) (2, 3)

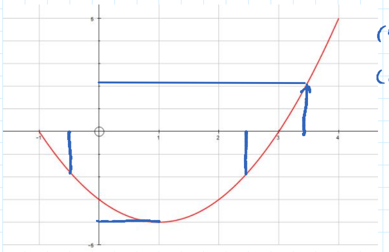


Bijective

Function = vertical
hit one = function.
Type = horizontal
Bijective.

Draw $f(x) = x^2-2x-3$ in domain $-1 \le x \le 4$ on a Cartesian Plane.

(−1, 0) (0, −3) (1, −4) (2, −3)
(3, 0) (4, 5)



(i) Function. Yes
(ii) Injective = many to one
(iii) Surjective. No not all the co-domain is used

Surjective if $y \ge -4$

$f : R \to R : x \to x^2-2x-3$
R = domain
R = codomain
$y \ge -4$ = range

Composition of Functions.

$f(x) = 5x+2$ $g(x) = 4x+1$
(i) Find $f(2)$ and then g(answer)
$f(2) = 12$

$g(12) = 49$

(ii) Find $g \circ f(3)$

$f(3) = 17$

$g(17) = 4(17) + 1 = 69$

$g \circ f(x) = g$ after f

(iii) $fg(5)$

$fg(5) = f(g(5)) = f \circ g(5)$

$g(5) = 21$

$f(21) = 5(21) + 2 = 107.$

$g(x) = 4x + 3 \qquad f(x) = x^2 - 5$

Find (i) $gf(x)$

(ii) $fg(x)$

(i) $gf(x) = g(f(x))$

$= g(x^2 - 5) = 4(x^2 - 5) + 3$

$= 4x^2 - 17$

(ii) $fg(x) = f(g(x))$

$g(x) = 4x + 3$

$f(4x+3) = (4x+3)^2 - 5$

$= 16x^2 + 24x + 9 - 5$

$= 16x^2 + 24x + 4$

In general $fg(x) \neq gf(x)$

Inverse $f^{-1}(x)$

$f : \mathbb{Z} \to \mathbb{Z} : x \to 2x - 1$ find range when domain is $\{-1, 0, 1\}$.

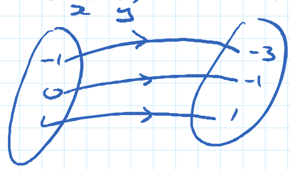
Show on arrow diagram.

Show $f^{-1}(x)$ on another arrow diagram.

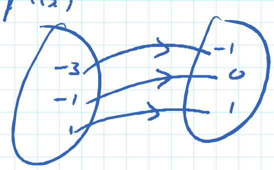
$(-1, -3) \quad (0, -1) \quad (1, 1)$

$x \ \ y$

$f(x)$



$f^{-1}(x)$



$(-3, -1) \quad (-1, 0) \quad (1, 1)$

$x \ \ y$

Inverse $\to$ bijective

$\to$ restrictions.

$f(x) = 5x + 7$ find $f^{-1}(x)$

$y = 5x + 7$ Find x

$5x + 7 = y$

$5x = y - 7$

$$x = \frac{y-7}{5}$$

$$f^{-1}(x) = \frac{y-7}{5}$$

Find $f^{-1}(x)$ of $y = \dfrac{3x-1}{8}$

$$y = \frac{3x-1}{8}$$

$$8y = 3x - 1$$

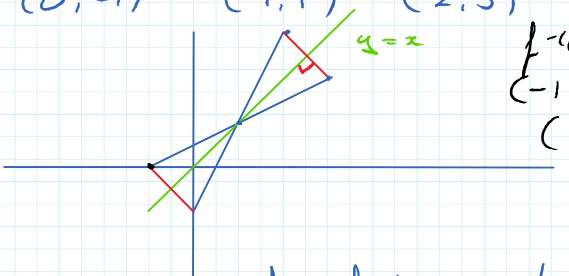
$$8y + 1 = 3x$$

$$\frac{8y+1}{3} = x$$

$$f^{-1}(x) = \frac{8x+1}{3}$$

Draw $f(x) = 2x-1$ in the domain $0 \leq x \leq 2$ and $f^{-1}(x)$ on same diagram

$(0,-1)$ $(1,1)$ $(2,3)$



$f^{-1}(x)$

$(-1,0)$ $(1,1)$ $(3,2)$

$f^{-1}(x)$ is image of $f(x)$ under axial symmetry in $y = x$.

Domain of $f(x)$ is range of $f^{-1}(x)$

$f(x) = x^2 - 6x + 3$ where $x \geq 3$ find $f^{-1}(x)$ and state domain of $f^{-1}(x)$.

$$y = x^2 - 6x + 3$$

$$y - 3 = x^2 - 6x$$

$$x^2 - 6x + 3 = y$$

$$x^2 - 6x + 9 + 3 - 9 = y$$

Minimum $(3,-6)$ $(x-3)^2 - 6 = y$

$$(x-3)^2 = y + 6$$

$$x - 3 = \sqrt{y+6}$$

$$x = \sqrt{y+6} + 3$$

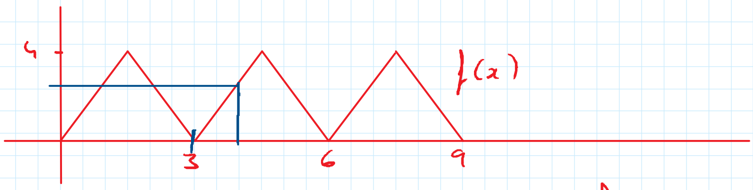
$$f^{-1}(x) = \sqrt{x+6} + 3 \qquad x > -6$$

$f(x) = x^2 - 12x + 3$ where $x > 6$

find $f^{-1}(x)$ state the domain.

$$y = x^2 - 12x + 3$$
$$y = x^2 - 12x + 36 + 3 - 36$$
$$y = (x-6)^2 - 33 \qquad (6, -33)$$
$$y + 33 = (x-6)^2$$
$$\sqrt{y+33} = x - 6$$
$$x = \sqrt{y+33} + 6$$
$$f^{-1}(x) = \sqrt{x+33} + 6$$
$$x > -33$$

Period and range.



(i) Period (iii) $f(253)$

(ii) Range

(i) Period = how quick it repeats

Period = 3

(ii) Range = [Low , High]

= [0, 4]

(iii) $f(253)$ $\frac{253}{3} = 84\frac{1}{3}$

$f(253) = 2.5$

Limits.

$$\lim_{x \to 2} (3x+1)$$

Lim is limit of $3x+1$

as x gets closer to 2.

$$\lim_{x \to 2} (3x+1) = 3(2)+1 = 7.$$

$$\lim_{x \to 1} (4x - x^2) = 4 - 1 = 3$$

$$\lim (4x - x^2) = 8 - 4 = 4$$

$x \to 2$

$\lim_{x \to 3} (4x - x^2) = 12 - 9 = 3$ ✓

$f(x) = 4x - x^2$

$y = 4x - x^2$

x = Time y = km

x = workers y = out

$\lim_{x \to \infty} \frac{1}{x} = \frac{1}{\infty}$

x	$\frac{1}{x}$
1	1
10	0.1
100	0.01
1000	0.001
$\downarrow$	$\downarrow$
∞	0

$$\boxed{\lim_{x \to \infty} \frac{1}{x} = 0}$$

$\lim_{x \to 0} \frac{1}{x} = \infty$.

x	$\frac{1}{x}$
1	1
0.1	10
0.01	100
0.001	1000
$\downarrow$	$\downarrow$
0	∞

$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \frac{0}{0} = \infty$ Factors.

$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$

$\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 5x + 6}$

$\lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-2)} = \frac{6}{1} = 6$

Simplify $\frac{3x + 12}{6x + 9}$

$= \frac{\frac{3x}{3} + \frac{12}{3}}{\frac{6x}{3} + \frac{9}{3}} = \frac{x + 4}{2x + 3}$

$$\lim_{x \to \infty} \frac{3x+1}{2x+5}$$

$x \to \infty$ - divide by highest power of x.

$$\lim_{x \to \infty} \frac{3 + \frac{1}{x}}{2 + \frac{5}{x}} = \frac{3}{2}$$

$$\lim_{x \to \infty} \frac{7x-3}{2x-5}$$

$$\lim_{x \to \infty} \frac{7 - \frac{3}{x}}{2 - \frac{5}{x}} = \frac{7}{2}$$

$$\lim_{x \to \infty} \frac{3x^2+5x}{2x^2+1}$$

$$\lim_{x \to \infty} \frac{3 + \frac{5}{x}}{2 + \frac{1}{x^2}} = \frac{3}{2}$$

$$\lim_{x \to \infty} \frac{1}{x^p} = 0$$

$$\lim_{x \to \infty} x^n = 0 \quad \text{where } |x| < 1$$