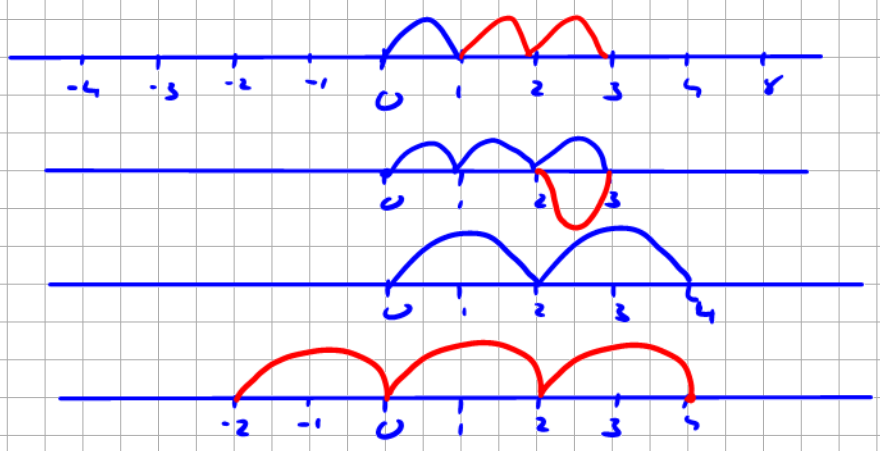
Complex Numbers

EKKER

Diagrams are important.



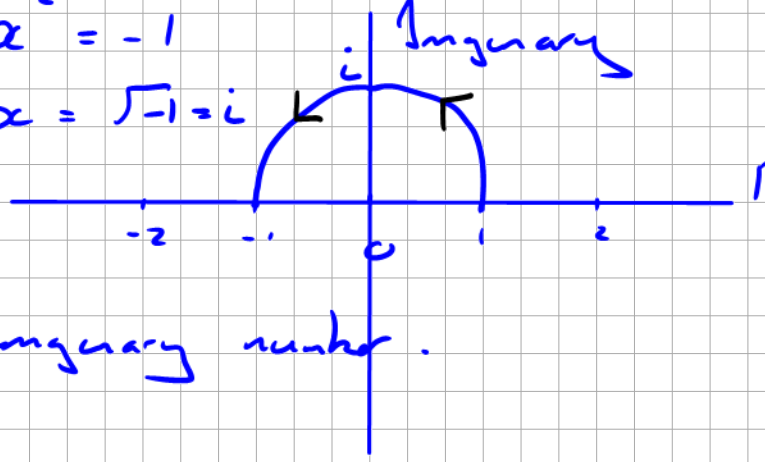
1 + 2
+ Right
3 - 1
- change
2(2) = 4
$(-2)^2$

$x^2 = 4$

$x = 2$
$x = -2$

$x^2 = -1$

$x = \sqrt{-1} = i$



i = iota = imgnary number.

Rules of Surds

(i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$

(ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a})^2 = a$

(iv) $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

Simplify

(i) $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

(ii) $\sqrt{-4} = \sqrt{4}\sqrt{-1} = 2i$

$$i = \sqrt{-1}$$
$$i^2 = -1$$

(iii) $\sqrt{-100} = \sqrt{100}\sqrt{-1}$

$10i$

Solve $x^2 + 2x + 5 = 0$ hence sketch

$y = x^2 + 2x + 5$

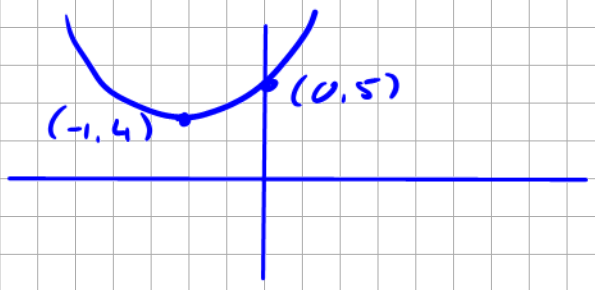
$a = 1 \quad b = 2 \quad c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 20}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$$

$\sqrt{-16} = \sqrt{16}\sqrt{-1} = 4i$

$$\frac{-2 \pm 4i}{2} = -1 \pm 2i$$



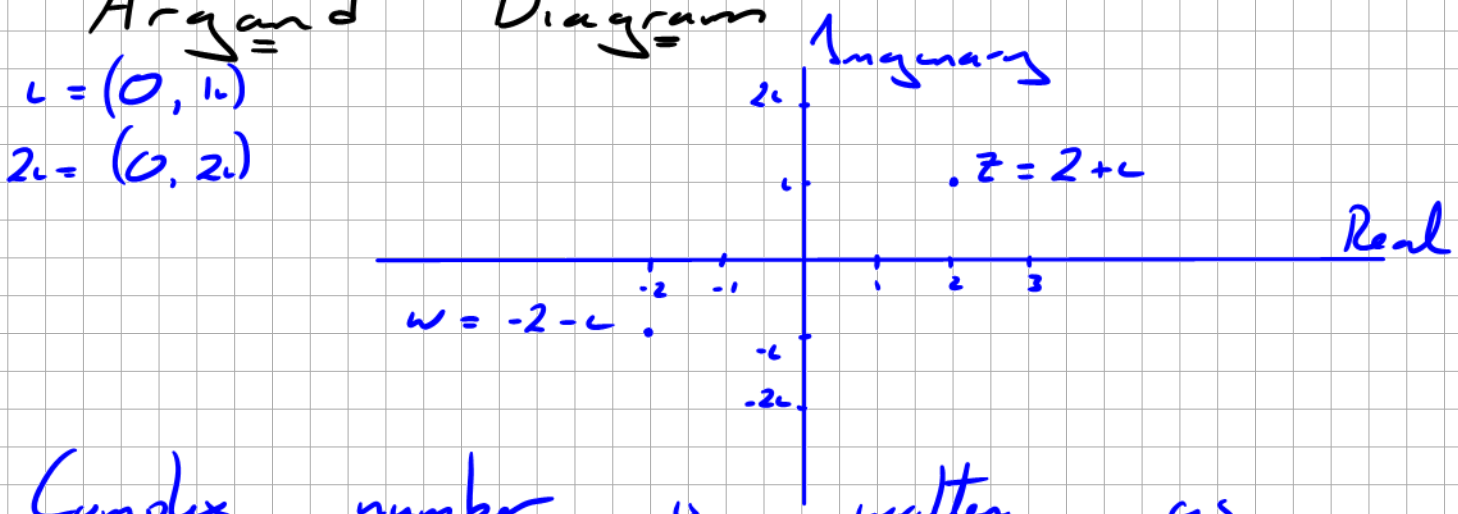
$y = x^2 + 2x + 5$

$y = x^2 + 2x + 1 + 5 - 1$

$y = (x+1)^2 + 4$

Basic of a Number System

Argand Diagram



Complex number is written as

$z = x + yi$ or (x, yi)

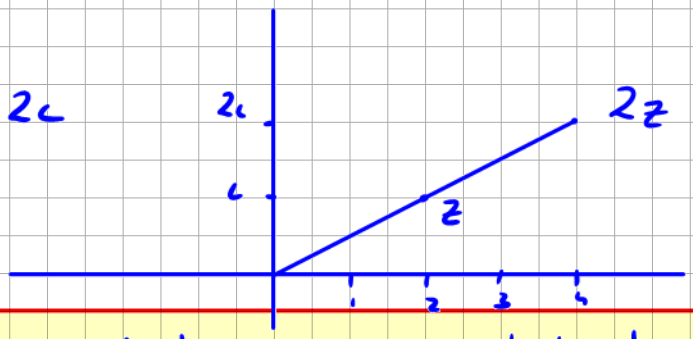
$x + yi$ is called rectangular.

(x, yi) means all co-ordinate geometry of line can be used. Pont format.

$z = 2 + i$ show z and $2z$ on Argand diagram.

$2 + i = (2, 1)$

$2(2 + i) = 4 + 2i$



$2z$ is called a dilation. It is an extension of line Oz. (Enlargement)

z and kz are always on same straight line from origin.

$k > 1 \Rightarrow$ out from z

$k < 1 \Rightarrow$ in toward origin.

$w = 2 + 3i$ find $3w$

$w = 2 + 3i$

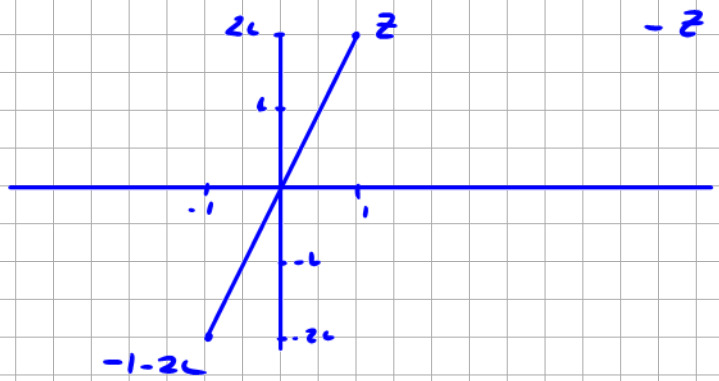
$3w = 3(2 + 3i)$

$= 6 + 9i$

$z = 1 + 2i$ plot on Argand diagram z and $-z$.

$-z = -(1 + 2i)$

$= -1 - 2i$



$-z$ is change direction of z.

(Central symmetry in origin).