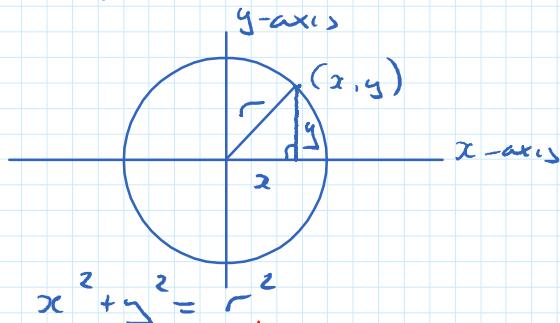


Circle centre  $(0,0)$



$$x^2 + y^2 = r^2$$

Write down centre and radius of

$$(i) \quad x^2 + y^2 = 25 \quad \text{Centre } (0,0) \quad r=5$$

$$(ii) \quad x^2 + y^2 = 40 \quad \text{Centre } (0,0) \quad r=\sqrt{40} \\ = 2\sqrt{10}$$

$$(iii) \quad 5x^2 + 5y^2 = 9$$

$$x^2 + y^2 = \frac{9}{5} \quad \text{Centre } (0,0)$$

Point inside, on or outside a Circle.

Is  $(-3, 7)$  inside, on or outside  $x^2 + y^2 = 47$ .

$$(-3)^2 + 7^2$$

$$9 + 49$$

$$58 > 47 \Rightarrow \text{outside}$$

Sub in point

$$\text{Ans} < r^2 \Rightarrow \text{inside}$$

$$\text{Ans} = r^2 \Rightarrow \text{on}$$

$$\text{Ans} > r^2 \Rightarrow \text{outside}$$

$(2, k)$  is inside  $x^2 + y^2 = 29$ .

Find range of values for  $k$ .

$$2^2 + k^2 < 29$$

$$\text{Centre } (0,0) \\ r = \sqrt{29}$$

$$4 + k^2 < 29$$

$$k^2 < 25$$

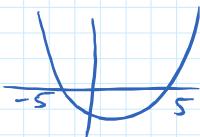
$$k^2 - 25 < 0$$

$$k^2 - 25 = 0$$

$$k^2 = 25$$

$$k = \pm 5$$

$$-5 < k < 5$$



Name 3 points inside  $x^2 + y^2 = 9$

$$x^2 + y^2 = 9 \quad \text{Centre } (0,0)$$

$$(1,0) \quad (0,1) \quad (2,0) \quad r=3$$

Cuts axes.

Find where  $x^2 + y^2 = 64$  cuts the x-axis.

$$x\text{-axis} \quad y = 0$$

$$x^2 = 64$$

$$x = \pm 8$$

$$(8,0) \quad (-8,0)$$

Tangent at a point.

Find tangent to  $x^2 + y^2 = 13$  at point  $(-2, 3)$ . Find parallel tangent.

$$x^2 + y^2 = 13$$

$$\text{Centre } (0,0)$$

$$(0,0)$$

$$r = \sqrt{13}$$

$$(0,0) \quad (-2,3)$$

$$m = \frac{3}{-2}$$

Required  $m = \frac{2}{3}$

$$y - 3 = \frac{2}{3}(x + 2)$$

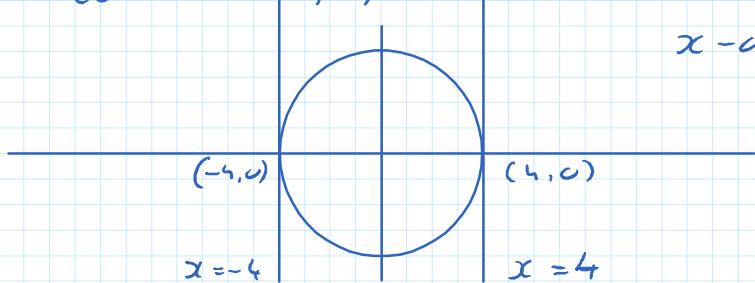
$$(-2,3) \rightarrow (0,0) \rightarrow (2,-3)$$

$$y + 3 = \frac{2}{3}(x - 2)$$

Find where  $x^2 + y^2 = 16$  cuts the x-axis.

State the tangent at these points

$$\text{Centre } (0,0) \quad r = 4$$



$$x\text{-axis} \quad y = 0$$

$$x^2 = 16$$

$$x = \pm 4$$

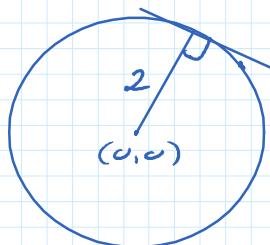
Tangents from outside a circle

Find tangents from  $(1, 3)$  to

$$x^2 + y^2 = 4.$$

$$r=2 \quad \text{Centre } (0,0)$$

$$(2,0) \quad (-2,0)$$



$$1^2 + 3^2 = 10 > 4$$

$\Rightarrow$  outside

$$\text{Line } \Rightarrow$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = m(x - 1)$$

Perpendicular distance from  $y - 3 = m(x - 1)$

to  $(0,0)$  is 2.

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$y - 3 = mx - m$$

$$mx - y + 3 - m = 0 \quad (0,0)$$

$$a = m \quad b = -1 \quad c = 3 - m \quad x_1 = 0 \quad y_1 = 0$$

$$\frac{|3-m|}{\sqrt{m^2+1}} = 2$$

$$|3-m| = 2\sqrt{m^2+1}$$

$$(3-m)^2 = 4(m^2+1)$$

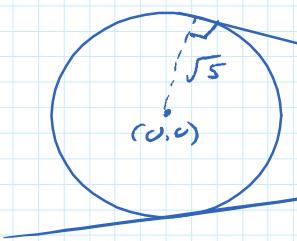
$$9 - 6m + m^2 = 4m^2 + 4$$

$$3m^2 + 6m - 5 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Find tangents to  $x^2 + y^2 = 5$   
from the point  $(5,0)$

$$\text{Centre } (0,0) \quad r = \sqrt{5}$$



$$25 > 5$$

$\Rightarrow$  outside

$$y - 0 = m(x - 5)$$

$$y = mx - 5m$$

$$mx - y - 5m = 0$$

$$a = m \quad b = -1 \quad c = -5m \quad x_1 = 0 \quad y_1 = 0$$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} =$$

$$\frac{|-5m|}{\sqrt{m^2 + 1}} = \sqrt{5}$$

$$|-5m| = \sqrt{5} \sqrt{m^2 + 1}$$

$$25m^2 = 5(m^2 + 1)$$

$$25m^2 = 5m^2 + 5$$

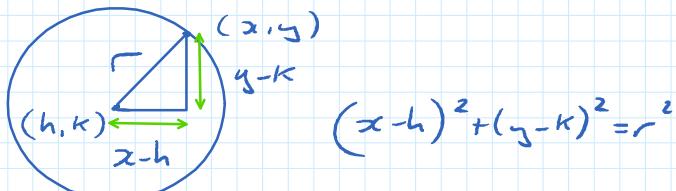
$$20m^2 = 5$$

$$m^2 = \frac{1}{4}$$

$$m = \pm \frac{1}{2}$$

$$y = \frac{1}{2}(x-5) \quad \text{or} \quad y = -\frac{1}{2}(x-5)$$

Circle centre  $(h, k)$



Find centre and radius of

$$(1) \quad (x-3)^2 + (y-5)^2 = 36$$

Centre  $(3, 5)$        $r = 6$

$$(2) \quad (x+5)^2 + (y-7)^2 = 49$$

Centre  $(-5, 7)$        $r = 7$

$$(3) \quad x^2 + (y-2)^2 = 8$$

Centre  $(0, 2)$   
 $r = \sqrt{8}$

$(x-3)^2 + (y+1)^2 = k$ . Given  $(2, -3) \rightarrow$   
on circle. Find

(i)  $k$ .

(ii) Tangent at  $(2, -3)$

(iii) Parallel tangent.

$$(2, -3)$$

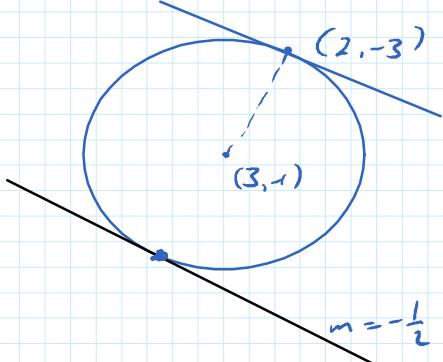
$\begin{matrix} x \\ \searrow \swarrow \end{matrix}$

$$(2-3)^2 + (-3+1)^2 = k$$

$$k = 5$$

Centre  $(3, -1)$

$$r = \sqrt{5}$$



$$(2, -3) \quad (3, -1)$$

$$m = \frac{-1+3}{3-2} = 2$$

$$\text{Required } m = -\frac{1}{2}$$

$$y+1 = -\frac{1}{2}(x-3)$$

$$(2, -3) \rightarrow (3, -1) \rightarrow (4, 1)$$

$$y-1 = -\frac{1}{2}(x-4)$$

Find where  $(x-3)^2 + (y+1)^2 = 25$   
cuts the  $y$ -axis.

$y$ -axis  $x = 0$

$$(-3)^2 + (y+1)^2 = 25$$

$$9 + (y+1)^2 = 25$$

$$(y+1)^2 = 16$$

$$y+1=4$$

$$y+1=-4$$

$$y=3$$

$$y=-5$$

$\therefore y = -5$ .