PI manual

A plumber charges €50 call out and €20 per hour. Show as table fees. Form a function for fees.

Time	Money	
0	50	
		+20
1	70	
		+20
2	90	
		+20
3	110	
x	$y = 20x + 50$	

50, 70, 90, 110
T_1 T_2 T_3 T_4

T_n = general term
= rule.

Sequence

$T_1, T_2, T_3, \ldots, T_{n-1}, T_n, T_{n+1}$

$T_n = U_n$ = rule

Series.

$$S_n = T_1 + T_2 + T_3 + T_4 + \cdots + T_n$$
$$= \sum_{n=1}^{n} T_n.$$

$$\sum_{n=1}^{\infty} 2n = 2 + 4 + 6 + \cdots = \infty$$

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots = 2 \; ?!$$

$T_n = 5n + 3$ find

(i) T_1

(ii) T_{10}

(iii) T_{n+1}.

(i) $T_1 = 5(1) = 8$

(ii) $T_{10} = 5(10) = 53$

(iii) $T_{n+1} = 5(n+1) + 3 = 5n + 8$

$T_n = 3^n + 5^n$ prove

$T_{n+2} - 8T_{n+1} + 15T_n = 0$

$T_n = 3^n + 5^n$

$T_{n+1} = 3^{n+1} + 5^{n+1}$

$= 3(3^n) + 5(5^n)$

$T_{n+2} = 3^{n+2} + 5^{n+2}$

$= 9(3^n) + 25(5^n)$

$x = 3^n \quad y = 5^n$

$9x + 25y - 8(3x+5y) + 15(x+y) = 0$

$T_n = 3T_{n-1}$ given $T_1 = 2$

find T_4.

$T_n = 3T_{n-1}$

$T_2 = 3T_1 = 3(2) = 6$

$T_3 = 3T_2 = 3(6) = 18$

$T_4 = \quad = 54$

$T_1 = T_2 = 1$ given

$T_{n+2} = T_{n+1} + T_n$ find T_5.

$T_3 = T_2 + T_1 = 1+1 = 2$

$T_4 = T_3 + T_2 = 2+1 = 3$

$T_5 = T_4 + T_3 = 3+2 = 5$

Recurring sequence.

To find T_n given S_n.

$S_{99} = 101$

$S_{100} = 98$ find T_{100}.

$S_{99} = T_1 + T_2 + \cdots + T_{99}$

$S_{100} = T_1 + T_2 + \cdots + T_{99} + T_{100}$

$T_{100} = S_{100} - S_{99}$

$= 98 - 100 = -3$

$S_n = n^2 + 7n$ find T_n.

$$T_n = S_n - S_{n-1}$$

$S_n = T_1 + T_2 + T_3 + \cdots + T_{n-1} + T_n$

$S_{n-1} = T_1 + T_2 + T_3 + \cdots + T_{n-1}$

$S_n - S_{n-1} = T_n$

$S_n = n^2 + 7n$

$S_{n-1} = (n-1)^2 + 7(n-1)$

$= n^2 - 2n + 1 + 7n - 7$

$= n^2 + 5n - 6$

$T_n = S_n - S_{n-1} = 2n + 6.$

Sequence.

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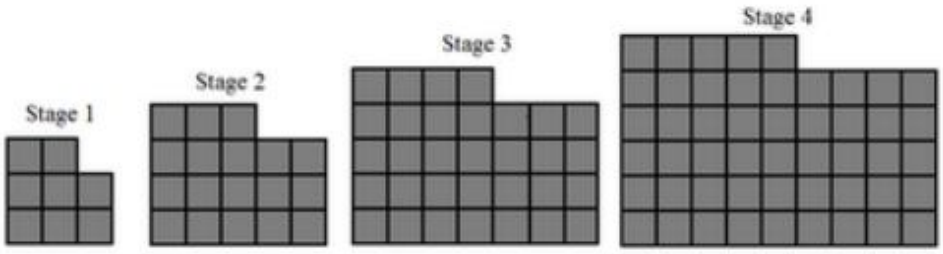

T_n

8, 18, 32, 50 find T_n.

10 14 18

4 4 ⇒ quadratic

$T_n = an^2 + bn + c$

Gap of gap $= 2 \Rightarrow a = 1$

$= 4 \Rightarrow a = 2$

$= 6 \Rightarrow a = 3$

$= k \Rightarrow a = \frac{k}{2}$

$T_n = 2n^2 + an + b$

$T_1 = 2 + a + b = 8$

$a + b = 6$

$T_2 = 2(4) + 2a + b = 18$

$2a + b = 10$

$a + b = 6$

$a = 4$

$b = 2$

$T_n = 2n^2 + 4n + 2$

$T_3 = 18 + 12 + 2 = 32$

Increasing / Decreasing.

Is $T_n = \frac{n+3}{n+1}$ increasing?

$n \in N$

Increasing $\Rightarrow$

$T_{n+1} > T_n$

$\frac{T_{n+1}}{T_n} > 1 \Rightarrow$ increasing

$T_n = \frac{n+3}{n+1}$

$T_{n+1} = \frac{n+4}{n+2}$

$n+2$

$$\frac{T_{n+1}}{T_n} = \frac{n+4}{n+2} \cdot \frac{n+1}{n+3}$$

$$= \frac{n^2+5n+4}{n^2+5n+6} < 1$$

$\Rightarrow$ decreasing

Convergent Sequence

$2, 4, 6, 8, 10, \dots$ Divergent

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$ Convergent

$\lim_{n\to\infty} T_n$ if exist $\Rightarrow$ convergent

$T_n = \frac{2n+1}{3n+5}$ is this convergent?

$$\lim_{n\to\infty} \frac{2n+1}{3n+5} = \lim_{n\to\infty} \frac{2+\frac{1}{n}}{3+\frac{5}{n}}$$

$$= \frac{2}{3} = \text{Convergent}$$

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

n	$\frac{1}{n}$
1	1
10	$\frac{1}{10}$
100	$\frac{1}{100} = 0.01$
	$= 0.001$

$= 0.001$

$$\lim_{n \to \infty} \frac{n^2 + 5n}{2n + 3}$$

$$\lim_{n \to \infty} \frac{1 + \frac{5}{n}}{\frac{2}{n} + \frac{3}{n^2}} = \frac{1}{0} = \infty$$

divergent.